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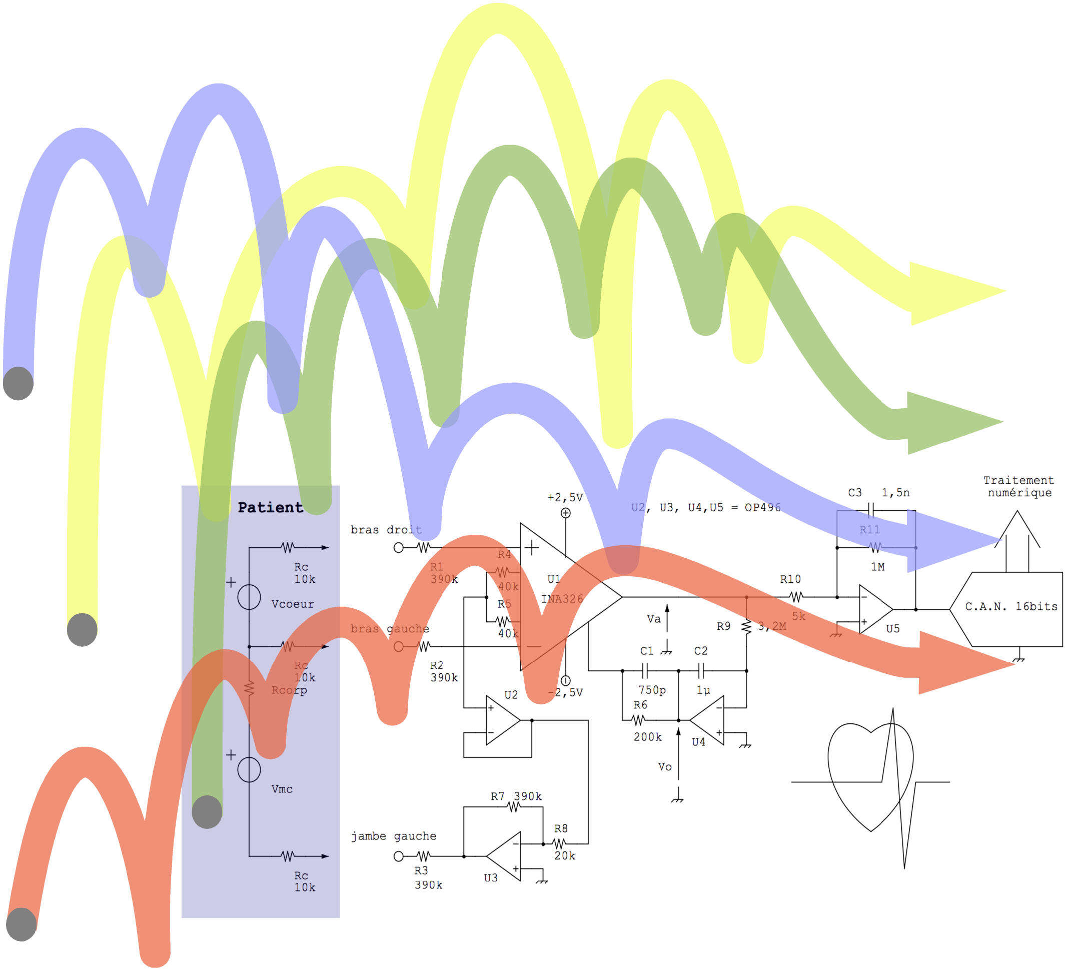
Amplifiers in measuring systems

Davide Bucci



Tronc commun PI

En couverture: interpretation libre basée sur une publicité de Giovanni Pintori pour Olivetti Lettera 22. Circuit simplifié de traitement de signaux pour éléctrocardiogramme.

CHAPTER 1

Amplification and amplifiers

1. Introduction

We described in [Bucci, 2021] a certain number of sensors, as well as the conditioning circuit used to obtain a voltage as an electrical representation of the measurand. Now, this voltage should be somehow treated: in most case it should be amplified, often filtered. This document is devoted to the amplification of low frequency signals and it is particularly focused on circuits based on operational amplifiers. We will therefore begin by briefly describing the working principles of operational amplifiers: in particular, we will focus on some parameters specified in the data-sheets quantifying their limits and defects. We will describe here the so called voltage-feedback operational amplifier (often just called operational amplifier), frequently adopted in low frequency circuits. A different element, the current-feedback operational amplifier bears some resemblance with it, but its use being more specific, it will not be described here. We will then give an overview of differential amplifiers, in particular the instrumentation amplifiers. The name of those circuits reflects their widespread use in instrumentation. . . The end of the document will be devoted to insulation amplifiers, precious when security or electromagnetic compatibility issues are of primary importance.

There are a lot of very good textbooks developing in detail the matter presented here. One of them is of course [Asch, 2003]. We recommend also the [Franco, 2002], which is very comprehensive and presents some advanced matters.

2. Introduction to operational amplifiers

2.1. The operational amplifier as a differential amplifier. First of all: an operational amplifier is an electronic circuit with two inputs and one output. It aims to be as close as possible to a *differential* amplifier with a very high gain. Figure 1 shows an idealised circuit representation of what we expect from an operational amplifier: it takes the voltage difference ϵ measured between the non inverting "+" and the inverting "-" inputs, it amplifies it and the amplified output voltage V_{o} is now referred to the reference node:

$$V_{\mathrm{o}} = A_{\mathrm{d}}\epsilon \tag{2.1}$$

The input is differential and the output is single ended. In practice, the differential gain A_{d} is very high, but its exact value strongly depends on the frequency and a variety of other factors.

Usually, an operational amplifier is drawn as shown in Figure 1, where the two inputs and the output is represented[1].

There is of course important point which has been left out: the power supply rails V_{CC} and V_{EE}, as shown in Figure 2 on the left. The same picture depicts a more realistic model of the operational amplifier, by taking into account the saturation: no signal can (at least in ordinary cases) exceed the power supply rails in an operational amplifier circuit. Note

[1]While the situation shown in Figure 1 is quite common in textbooks, it is evidently a strongly idealised one, or in any case it can not be complete. In fact, *how can the operational refer its output to the reference node if it does not have any other connection to it?* This means that something must have been left out in the drawing and, for example, a lot of SPICE models for commercial operational amplifiers contain an artificial internal connection to the reference node. Be careful with SPICE simulations: believing that a result is accurate only because it comes from a computer is usually a good way to seek for catastrophes.

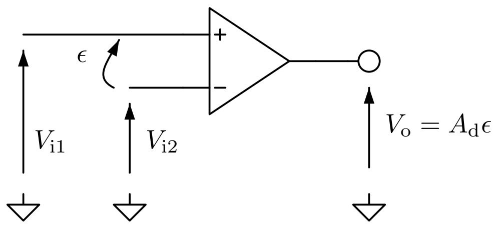


FIGURE 1. An operational amplifier as a differential amplifier.

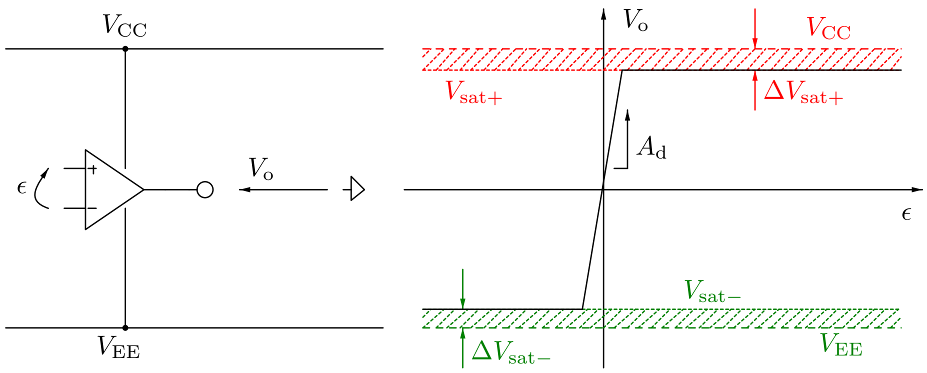


FIGURE 2. On the left: an operational amplifier with explicit representation of the power supply rails. On the right, a graphical representation of the (idealised) output characteristics.

that in the picture we represented the power supply rails symmetrically with respect to the reference potential. With A_d ranging usually between 10^5 and 10^6 (100 to 120 dB) at DC, if one wants to exploit the linear part of the characteristics, the only way is to use feedback. In other words, one must provide a link between the output and the input so that the output can be adjusted by the amplifier so that the differential voltage ϵ at the inputs is always in the linear range of the amplifier.

2.2. Modeling ideal operational amplifiers. Since the differential gain A_d is so high and moreover practically uncontrolled, we can say that *an operational amplifier is a circuit (often quite complex) optimised to be used as a linear block, by means of a feedback network.* A very useful model allows to simplify the calculations on the circuits by following two rules:

R1: The amplifier is able to measure a differential voltage without perturbing the circuit: no current flows in the inputs.

R2: The differential gain A_d is considered infinite. Therefore the only admissible situation where V_o is limited is $\epsilon = 0$. We may formulate that by saying that the amplifier does whatever it can with its output, to make sort that its inputs remain at the same voltage.

We notice that this way of seeing things is based on the presence of a feedback: otherwise, the output can not affect the inputs. We also remark that this simple model does not make a difference between the inverting and the non-inverting inputs, while we know that they can not be exchanged in a real circuit: saturation inevitably occurs if a confusion is done.

The rules associated to ideal operational amplifiers allow to understand the basic behaviours of a circuit. However, many key characteristics may only be deduced by taking into account more realistic models. In the following paragraphs, we will therefore briefly describe some of the most relevant defects of real operational amplifiers. Knowing their influence in a circuit allows to understand the data-sheets and select the best product in the huge catalogues proposed by the semiconductor companies.

3. Limits of real operational amplifiers

3.1. Saturation and rail to rail operational amplifiers. In figure 2, we notice that the output voltage of the amplifier is bounded inside a range defined by $V_{\mathrm{sat}+}$ and $V_{\mathrm{sat}-}$. One significant figure of merit of the amplifier is the difference between the saturation voltages and the power supply rails ($\Delta V_{\mathrm{sat}+}$ and $\Delta V_{\mathrm{sat}-}$), reflecting the capability of the circuit of working close to V_CC and V_EE. In classic integrated operational amplifiers $\Delta V_{\mathrm{sat}+}$ and $\Delta V_{\mathrm{sat}-}$ were between 1 and 2 volts. Today, we assist to the widespread use of wireless devices containing batteries, as well to a general trend coming from the digital circuits of reducing power supply voltage. Such a margin would represent a huge reduction of the available dynamic range. In fact, it is not uncommon to seek for high performance analog circuits with a single $V_\mathrm{CC} - V_\mathrm{EE} = 3.3\,\mathrm{V}$ supply or even less. For this reason, a class of operational amplifiers (called rail to rail) has been optimised to make sort that $\Delta V_{\mathrm{sat}\pm}$ do not exceed one hundred millivolts in the operating conditions.

What we described above is referred in particular to the output section of the operational amplifier, but something similar also happens at the inputs: most amplifiers do not work well if the voltages at their inputs are too close to V_CC and V_EE. Some of the modern rail to rail amplifiers, however, tolerate both inputs to be slightly above V_CC or below V_EE, adding flexibility in single supply configurations.

3.2. Input offset. For several reasons (mainly some small asymmetries in the fabrication process), when the voltage applied to the two inputs of a real operational amplifier is equal, the output voltage is not zero as predicted by equation (2.1). In fact, the very high value of the differential DC gain A_d will probably make sort that the asymmetry is such exasperated that output is saturated, either at $V_{\mathrm{sat}+}$ or $V_{\mathrm{sat}-}$. A small DC voltage, called the *offset voltage*, should be applied between the inputs in order that the output is no longer in this condition. The offset voltage can range between some microvolts in precision operational amplifier, to several millivolts. This effect being static, it affects only those circuits whose bandwidth include DC.

When feedback is present, the presence of offset changes Rule 2 in such a way that the difference between input voltages is no longer null, but equal to the value of the offset voltage. External nulling can often be performed via an external adjustable resistive network.

3.3. Common mode rejection ratio. In an ideal differential amplifier, the output voltage depends only by the voltage difference between the two inputs, that we called ϵ in fig. 1. In a real device, this is not completely true and the average of the two inputs voltages (V_i1 and V_i2) plays a small role. In other words, by supposing that only this defect is present, equation (2.1) should be corrected as follows:

$$V_\mathrm{o} = A_\mathrm{d}\epsilon + A_\mathrm{cm}V_\mathrm{cm} \tag{3.1}$$

where $V_{\rm cm}$ is the so-called common mode, i.e. the arithmetic average of the voltages at the two inputs of the amplifier, each one referred to the reference node. To summarise:

$$\begin{cases} \epsilon = V_{\rm i1} - V_{\rm i2} \\ V_{\rm cm} = \frac{V_{\rm i1}+V_{\rm i2}}{2} \end{cases} \tag{3.2}$$

A new term of gain, namely $A_{\rm cm}$, the common mode gain appears. A good differential amplifier (and thus a good operational amplifier) should make sort that $A_{\rm d}$ is much greater than $A_{\rm cm}$. To quantify this characteristics, the data-sheets report the *common mode rejection ratio* in decibel, defined as follows:

$$C_{\rm mrr} = 20 \log \frac{A_{\rm d}}{A_{\rm cm}} \tag{3.3}$$

where the logarithm is base 10. Typical figures range between 80 to 120 dB.

3.4. Bias currents. In paragraph 2.2, the First rule states that no current flows in the inputs of an ideal operational amplifier. In real circuits, things are different: *some* current must flow to in the inputs, since a voltage measurement must be done. It is desirable to keep it as small as possible and operational amplifiers have been vastly optimised in this regard: currents as low as several picoampers are not uncommon in modern devices.

3.5. Stability and frequency response. In our context, we call a circuit stable when the output to a bounded input is bounded. Other different definitions of stability exist. This definition is usually called with the acronym BIBO, from Bounded In Bounded Out. It is often highly desirable that a circuit remains stable. In low power applications, like operational amplifier circuits, the lack of stability will show up with non linearity, saturations, parasitic oscillations and head-scratching problems. In high power applications, lack of stability may yield expensive repairs, safety hazards, fires, explosions, nuclear meltdowns... If a circuit is unstable, most of the times it is practically useless.[2]

In modern voltage-feedback operational amplifiers there is an internal compensation network which tries to sort out a trade-off between overall speed and stability. The need of stability makes sort that the small signal bandwidth of the operational amplifier is often limited by introducing a low frequency dominant pole in the differential gain $A_{\rm d}$. A consequence of that is that when a feedback network is present to obtain a circuit having a controlled gain G, the product $Gf_{\rm p}$ is approximatively constant, where $f_{\rm p}$ is the bandwidth obtained by the circuit. In other words, increasing the gain by acting on the feedback around the same operational amplifier entails a reduction of the frequency band treatable by the circuit. There is however a notable case in which the designer needs to take special care: most of operational amplifiers do not appreciate capacitive loads at their output (capacitors, long cables etc.).

If the limitation of the bandwidth is an effect associated to the small signal behaviour of the circuits, but a large signal (nonlinear) effect is also evident: there is a limitation to the slope of the variation of the output voltage versus the time (the so called *slew rate*).

3.6. Examples. Figure 3 shows an extract of the data-sheet of an operational amplifier optimised for low bias current. We notice the typical bias current value of 3 fA at a temperature of 25 °C, which tends to rise with the temperature (5 pA at 125 °C is a good achievement). Note also how the output saturation voltages are clearly specified: this is a rail to rail operational amplifier and this stuff matters! Its performances in terms of noise, gain-bandwidth product and slew-rate are also quite sound.

Figure 3 does not show the input offset voltage, specified elsewhere to be typically $\pm 50\,\mu$V at 25 °C and less than $\pm 480\,\mu$V over an extended temperature range. This is a very decent offset performance, but it is clear that the device is not optimised towards this direction. As

[2]With the notable exception of oscillators, where a certain degree of instability is sought and kept under close control in order to initiate and sustain the oscillation.

TEXAS INSTRUMENTS **LMP7721**

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2.5V Electrical Characteristics (continued)

Unless otherwise specified, all limits are specified for T_A = 25°C, V^+ = 2.5V, V^- = 0V, V_{CM} = (V^+ + V^-)/2. **Boldface** limits apply at the temperature extremes.

Symbol	Parameter	Conditions		Min (1)	Typ (2)	Max (1)	Units
I_{BIAS}	Input Bias Current	V_{CM} = 1V (4) (5)	25°C		±3	±20	fA
			−40°C to 85°C			±900	
			−40°C to 125°C			**±5**	pA
I_{OS}	Input Offset Current	V_{CM} = 1V (5)			6	40	fA
CMRR	Common Mode Rejection Ratio	0V ≤ V_{CM} ≤ 1.4V		83 **80**	100		dB
PSRR	Power Supply Rejection Ratio	1.8V ≤ V^+ ≤ 5.5V V^- = 0V, V_{CM} = 0		84 **80**	92		dB
CMVR	Input Common-Mode Voltage Range	CMRR ≥ 80 dB CMRR ≥ 78 dB		−0.3 **−0.3**		1.5 **1.5**	V
A_{VOL}	Large Signal Voltage Gain	V_O = 0.15V to 2.2V R_L = 2 kΩ to $V^+/2$		88 **82**	107		dB
		V_O = 0.15V to 2.2V R_L = 10 kΩ to $V^+/2$		92 **88**	120		
V_O	Output Swing High	R_L = 2 kΩ to $V^+/2$		70 **77**	25		mV from V^+
		R_L = 10 kΩ to $V^+/2$		60 **66**	20		
	Output Swing Low	R_L = 2 kΩ to $V^+/2$			30	70 **73**	mV
		R_L = 10 kΩ to $V^+/2$			15	60 **62**	
I_O	Output Short Circuit Current	Sourcing to V^- V_{IN} = 200 mV (6)		36 **30**	46		mA
		Sinking to V^+ V_{IN} = −200 mV (6)		7.5 **5.0**	15		
I_S	Supply Current				1.1	1.5 **1.75**	mA
SR	Slew Rate	A_V = +1, Rising (10% to 90%)			9.3		V/μs
		A_V = +1, Falling (90% to 10%)			10.8		
GBW	Gain Bandwidth Product				15		MHz
e_n	Input-Referred Voltage Noise	f = 400 Hz			8		nV/$\sqrt{Hz}$
		f = 1 kHz			7		
i_n	Input-Referred Current Noise	f = 1 kHz			0.01		pA/$\sqrt{Hz}$
THD+N	Total Harmonic Distortion + Noise	f = 1 kHz, A_V = 2, R_L = 100 kΩ V_O = 0.9 V_{PP}			0.003		%
		f = 1 kHz, A_V = 2, R_L = 600Ω V_O = 0.9 V_{PP}			0.003		

(4) Positive current corresponds to current flowing into the device.
(5) This parameter is specified by design and/or characterization and is not tested in production.
(6) The short circuit test is a momentary open loop test.

FIGURE 3. Some of the characteristics of the LMP7721 operational amplifier, from Texas Instruments.

an exercise, compare this device with those in the following list (search for the data-sheet by yourself):

- The venerable general purpose bipolar μA741, designed in 1968 but still produced today. Compare it with the 1967 vintage LM101, today almost forgotten. Why μA741 was so successful?
- The JFET-input TL081.
- The first precision bipolar operational amplifier OP07.

Do not forget to search those amplifiers in the online catalogue of your favourite electronics dealer. Compare their costs.

4. Instrumentation amplifiers

4.1. Introduction. In the document [Bucci, 2021], we saw how the output of a sensor conditioning circuit is a voltage, which most of the times needs to be amplified. Moreover, in some situations, the voltage signal carrying information is not single ended (i.e. referred

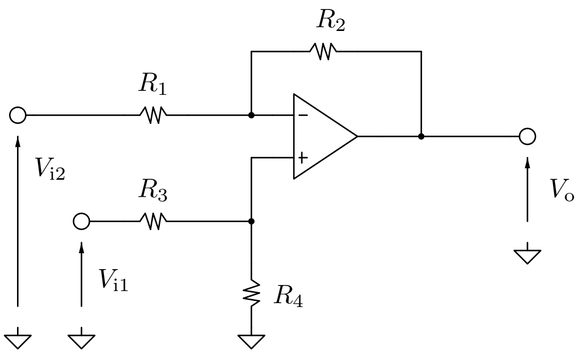


FIGURE 4. Differential amplifier built using one operational amplifier.

to the reference node) but differential. A classical example is the Wheatstone bridge with resistive sensors: the output signal is a voltage difference between two nodes.

We thus need a circuit able to extract the differential voltage signal without perturbing it, thanks to a high input impedance. We would also like to be able to easily adjust the differential gain of the circuit in a reasonable range, by changing only one component of the circuit. The ability of extracting the differential voltage regardlessly of the common mode voltage is quantified by the common mode rejection ratio parameter which is desirably very high. In the following paragraphs, we will describe a selection of classical circuits to achieve this goal, all based on operational amplifiers.

4.2. Differential amplifier with one operational amplifier. Figure 4 shows the classical differential amplifier made with one operational amplifier. The idea is to apply some amount of feedback in order to tame the the differential gain of the operational amplifier (as you remember from paragraph 2.1: it is very high, but it is variable, as affected by the frequency, power supply voltage, operating temperature etc.).

A little bit of circuit analysis, applying the rules given in paragraph 2.2, allows to write down the relation between voltages at the inputs V_{i1} and V_{i2} and the output V_o:

$$V_o = \frac{R_1 + R_2}{R_1} \times \frac{R_4}{R_3 + R_4} V_{i1} - \frac{R_2}{R_1} V_{i2}. \tag{4.1}$$

To understand how this circuit can be exploited as a differential amplifier, we rewrite this expression by representing the electrical state of the inputs using the differential and common mode voltages. Thus, we apply the following relations, which can be seen as some sort of a coordinates change:

$$\begin{cases} V_d = V_{i1} - V_{i2} \\ V_{cm} = \frac{V_{i1} + V_{i2}}{2} \end{cases} \tag{4.2}$$

V_d is the differential mode (the signal carrying the information to be extracted) and V_{cm} is the common mode. By inverting the relations, we obtain:

$$\begin{cases} V_{i1} = V_{cm} + \frac{V_d}{2} \\ V_{i2} = V_{cm} - \frac{V_d}{2} \end{cases} . \tag{4.3}$$

This yields expressions of V_{i1} and V_{i2} to be injected in equation (4.1) to obtain equation (4.4). It relates the output voltage (single ended) to the input differential and common modes of the voltages:

$$V_o = \frac{R_1 R_4 - R_2 R_3}{R_1(R_3 + R_4)} V_{cm} + \frac{R_1 + R_2}{2R_1} \left(\frac{R_4}{R_3 + R_4} + \frac{R_2}{R_1 + R_2} \right) V_d. \tag{4.4}$$

In the expression (4.4) we recognise the contribution of the differential gain as well as the common mode gain:

$$A_{\text{cm}} = \frac{R_1 R_4 - R_2 R_3}{R_1(R_3 + R_4)}, \tag{4.5}$$

$$A_{\text{d}} = \frac{R_1 + R_2}{2R_1}\left(\frac{R_4}{R_3 + R_4} + \frac{R_2}{R_1 + R_2}\right). \tag{4.6}$$

If a perfect differential amplifier has to be built, resistances $R_1 \ldots R_3$ should be chosen in such a way that the common mode gain is equal to zero. This can be achieved by nulling the numerator of the expression (4.5), thus giving:

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} \tag{4.7}$$

leading to a simple expression for the differential gain:

$$A_{\text{d}} = \frac{R_2}{R_1}. \tag{4.8}$$

In practice, very often $R_1 = R_3$ and $R_2 = R_4$, yet perfectly achieving this condition is not possible, because of the inevitable tolerance of the resistances (this takes into account the effect of ageing and thermal drift). In practice, we know that every resistance has a certain relative shift from its nominal value. We suppose that the following conditions are verified (worst case scenario):

- The relative shift of the value of each resistance is equal to the tolerance r.
- The shifts are distributed in such a way that the common mode gain A_{cm} is maximised:

$$\begin{cases} R_1 = R_{1\text{n}}(1 + r) \\ R_3 = R_{1\text{n}}(1 - r) \\ R_2 = R_{2\text{n}}(1 - r) \\ R_4 = R_{2\text{n}}(1 + r) \end{cases} \tag{4.9}$$

  where $R_{1\text{n}}$ and $R_{2\text{n}}$ are the nominal values matching condition equation (4.7).

We obtain that the common mode gain is not zero, and it is proportional to the tolerance r:

$$A_{\text{cm}} = \frac{4rR_{2\text{n}}}{R_{1\text{n}} + R_{2\text{n}}}. \tag{4.10}$$

To calculate the common mode rejection ratio, we suppose that the differential gain has not changed very much if r is small, yielding:

$$C_{\text{mrr}} = 20 \log_{10} \frac{A_{\text{d}}}{A_{\text{mc}}} \approx 20 \log_{10} \frac{R_{1\text{n}} + R_{2\text{n}}}{4rR_{1\text{n}}}. \tag{4.11}$$

In the worst case scenario, this means that by adopting $r = 0.1\%$ tolerance for the resistances, by choosing a gain $A_{\text{d}} = 100$, we might expect that the common mode rejection ratio is about 88 dB. This circuit has some defects:

- The input impedances are proportional to the values of the resistances. Very high resistance values are however associated to noise and the resulting low currents might be sensitive to stray capacitances, couplings…
- The gain can be modified, but the relation (4.7) should be respected. At least two matched resistances should be varied at the same time to vary the differential gain without disrupting the differential behaviour.

To solve the first problem, a second operational amplifier can be added, as discussed in the next paragraph.

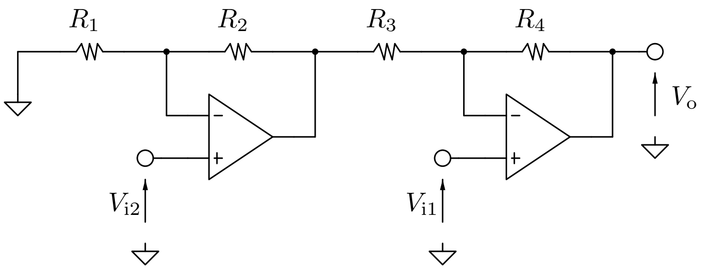


FIGURE 5. Differential amplifier built using two operational amplifiers.

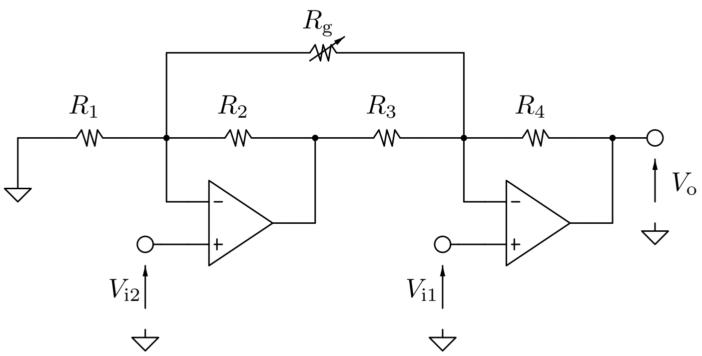


FIGURE 6. Differential amplifier with variable gain.

4.3. Differential amplifier with two operational amplifiers. A useful way to vastly increase input impedances is to exploit the excellent input characteristics of operational amplifiers. The circuit shown in figure 5 solves the first issue seen at the end of paragraph 4.2, namely the low input impedances. In the circuit, the two inputs are directly connected to the inputs of the operational amplifiers. For this reason, once the correct biasing of the operational amplifiers is assured, inputs are extremely high impedance.

Analysing the circuit in the way described in paragraph 4.2, we calculate the differential gain, as well as the common mode gain:

$$\begin{cases} A_\mathrm{d} = \frac{1}{2}\left[1 + \frac{R_4}{R_3}\left(2 + \frac{R_2}{R_1}\right)\right] \\ A_\mathrm{cm} = \left[\frac{R_4+R_3}{R_3} - \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right)\right] . \end{cases} \tag{4.12}$$

Nulling the latter, we obtain the balance condition of resistances $R_1 \ldots R_4$ to be respected:

$$\frac{R_1}{R_2} = \frac{R_4}{R_3} \tag{4.13}$$

thus yielding a simplified expression for the differential gain when the amplifier is purely differential:

$$A_\mathrm{d} = 1 + \frac{R_1}{R_2} \tag{4.14}$$

At a first sight, it might seem that this circuit is unable to solve the second problem described in the previous paragraph, i.e. the fact that it might not be easy to change the gain by modifying two matched resistances at the same time. In reality, a solution exists connecting a fifth resistor R_g, adjustable, which allows to trim the gain without bothering with two matched devices, as shown in figure 6. In this case, the ratio described by equation (4.13)

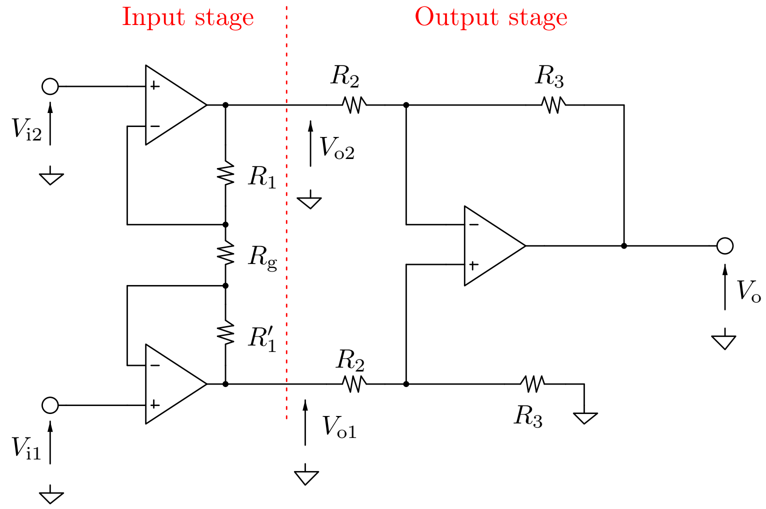


FIGURE 7. Differential amplifier with three operational amplifier: the instrumentation amplifier by antonomasia.

must be respected, but the differential gain can be written as follows:

$$A_d = 1 + 2\frac{R_1}{R_g} + \frac{R_1}{R_2} \tag{4.15}$$

If the two issues of the differential amplifier discussed in paragraph 4.2 have been successfully addressed, this circuit still has a more subtile flaw. In fact, the signal paths are not symmetrical: the signal entering from V_{i1} passes through two operational amplifiers, whereas the signal entering from V_{i1} just have to pass through one. When the limitations of the operational amplifiers begin to play an important role (for example, when the frequency is relatively high), the asymmetry decreases performances and in particular the common mode rejection ratio of the circuit.

4.4. Differential amplifier with three operational amplifiers. The circuit shown in figure 7 is a more complex differential amplifier. It is quite commonly used in instrumentation chains and for this reason, when people say "instrumentation amplifier", they are often referring to this particular circuit. To understand its behaviour, we split it in two sub-circuits:

- An input stage, which has a differential input and a differential output, meant to boost the differential mode, while leaving untouched the common mode.
- A differential amplifier, to provide a single ended output related to the input differential mode.

The second stage is in fact the circuit discussed in paragraph 4.2, so we analyse now the input stage as shown in figure 8, which is perfectly symmetrical if $R_1 = R_1'$. If we suppose that the operational amplifiers are ideal, rule 2 seen in paragraph 2.2 states that the voltages at the nodes A and B are equal respectively to V_{i2} and V_{i1}. By supposing for a moment that V_{o1} and V_{o2} are known, we apply the Millman theorem:

$$\begin{cases} \text{node A: } \dfrac{\frac{V_{o2}}{R_1} + \frac{V_{i1}}{R_g}}{\frac{1}{R_1} + \frac{1}{R_g}} = V_{i2} \\ \text{node B: } \dfrac{\frac{V_{i2}}{R_g} + \frac{V_{o1}}{R_1'}}{\frac{1}{R_g} + \frac{1}{R_1'}} = V_{i1} \end{cases} \tag{4.16}$$

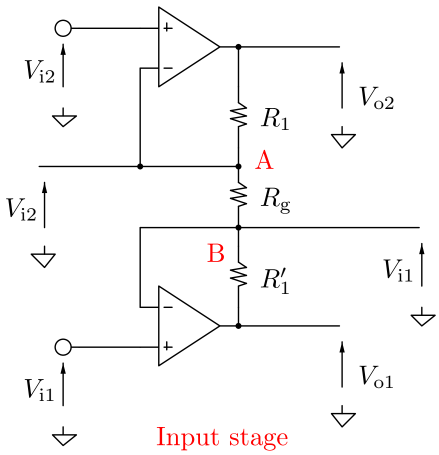


FIGURE 8. The symmetrical input stage of the instrumentation amplifier shown in figure 7.

By rearranging terms, we get:

$$\begin{cases} V_{o1} = \frac{R_g+R_1'}{R_g} V_{i1} - \frac{R_1'}{R_g} V_{i2} \\ V_{o2} = \frac{R_g+R_1}{R_g} V_{i2} - \frac{R_1}{R_g} V_{i1}. \end{cases} \tag{4.17}$$

Similarly to what done for the input signals with equations (4.2) and (4.3), we define differential and common mode voltages for the two outputs V_{o1} and V_{o1}:

$$\begin{cases} V_d' = V_{o1} - V_{o2} \\ V_{cm}' = \frac{V_{o1}+V_{o2}}{2}. \end{cases} \tag{4.18}$$

However, inevitable tolerances make sort that R_1 and R_1' are not identical. We consider the worst case scenario, as follows:

$$\begin{cases} R_1 = R_{1n}(1+r) \\ R_1' = R_{1n}(1-r) \end{cases} \tag{4.19}$$

where r is the tolerance of the resistances, whose nominal value is R_{1n}. In those conditions, we relate the output common mode with the input common and differential modes. After some algebra, we obtain:

$$V_{cm}' = V_{cm} + \frac{R_{1n}}{R_g} r V_d \tag{4.20}$$

as well as the output differential mode:

$$V_d' = \left(1 + \frac{2R_{1n}}{R_g}\right) V_d. \tag{4.21}$$

We notice that:

- If R_g is much smaller than R_{1n}, the input differential mode is greatly amplified.
- The input common mode is mostly not amplified nor attenuated by the first part of the circuit. This contribution is very often the most relevant one to the output common mode voltage.
- The output common mode is also affected by the input differential voltage, in a factor which is dependent on the R_{1n}/R_g ratio (the same affecting the differential gain) as well as the tolerance r of the matching between R_1 and R_1'.

To summarise, the first circuit has a differential and common mode gains A'_{d} and A'_{cm} as follows:

$$\begin{cases} A'_{\mathrm{d}} = 1 + \frac{2R_{1\mathrm{n}}}{R_{\mathrm{g}}} \\ A'_{\mathrm{cm}} \approx 1 \end{cases} . \tag{4.22}$$

Paragraph 4.2 presented the analysis of the second half of the circuit of figure 7:

$$\begin{cases} A''_{\mathrm{mc}} = \frac{4rR_{3\mathrm{n}}}{R_{2\mathrm{n}}+R_{3\mathrm{n}}} \\ A''_{\mathrm{d}} = \frac{R_{3\mathrm{n}}}{R_{2\mathrm{n}}} \end{cases} , \tag{4.23}$$

where as usual r represents the tolerance of the resistances and the "n" subscript indicates their nominal values.

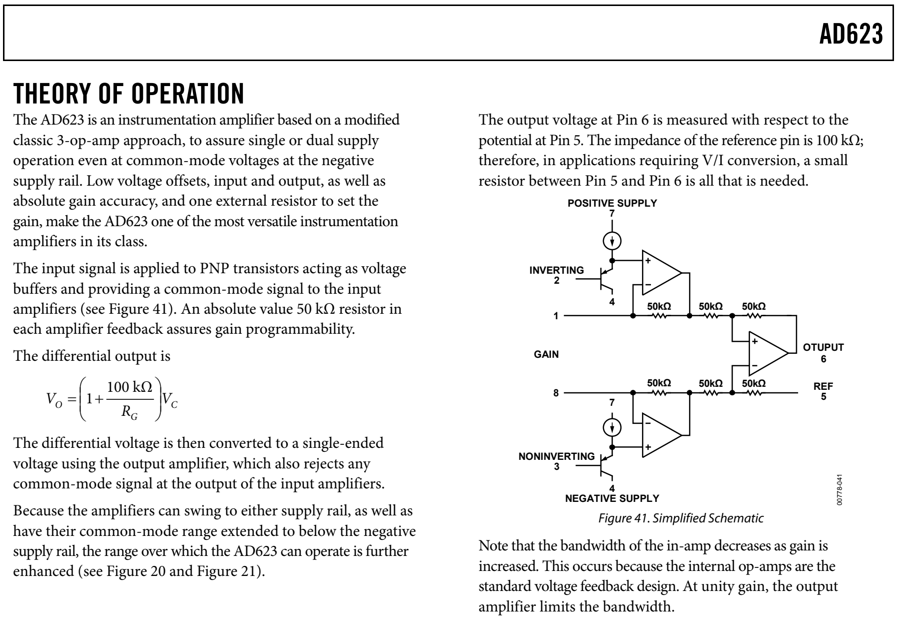
Putting together all these equations (and neglecting some cross terms) yields the differential and common mode gain of the complete amplifier:

$$\begin{cases} A_{\mathrm{mc}} = A'_{\mathrm{mc}}A''_{\mathrm{mc}} \approx \frac{4rR_{3\mathrm{n}}}{R_{2\mathrm{n}}+R_{3\mathrm{n}}} \\ A_{\mathrm{d}} = A'_{\mathrm{d}}A''_{\mathrm{d}} = \frac{R_{3\mathrm{n}}}{R_{2\mathrm{n}}}\left(1 + \frac{2R_{1\mathrm{n}}}{R_{\mathrm{g}}}\right) \end{cases} . \tag{4.24}$$

Those equations might be furthermore simplified when $R_{3\mathrm{n}} = R_{2\mathrm{n}}$, which is a frequent choice:

$$\begin{cases} A_{\mathrm{mc}} \approx 2r \\ A_{\mathrm{d}} = A'_{\mathrm{d}}A''_{\mathrm{d}} = 1 + \frac{2R_{1\mathrm{n}}}{R_{\mathrm{g}}} \end{cases} . \tag{4.25}$$

In fact, the instrumentation amplifier built around 3 operational amplifiers is both flexible and very convenient to be integrated (for example the INA101, the AD623 and the INA333 and many others). In fact, in microelectronics it is difficult to control precisely the absolute value of a passive device, but symmetries such as those required in this circuit can be achieved quite conveniently. In fact, the end user just needs to choose the gain via the resistance R_{g} which is normally to be connected outside of the integrated circuit. This both provides outstanding performances, ease to use as well as flexibility. For example, have a look at figure 9, where the AD623 is described. Compare the expression given for the gain with equation (4.25).



AD623

THEORY OF OPERATION

The AD623 is an instrumentation amplifier based on a modified classic 3-op-amp approach, to assure single or dual supply operation even at common-mode voltages at the negative supply rail. Low voltage offsets, input and output, as well as absolute gain accuracy, and one external resistor to set the gain, make the AD623 one of the most versatile instrumentation amplifiers in its class.

The input signal is applied to PNP transistors acting as voltage buffers and providing a common-mode signal to the input amplifiers (see Figure 41). An absolute value 50 kΩ resistor in each amplifier feedback assures gain programmability.

The differential output is

$$V_O = \left(1 + \frac{100\ \text{k}\Omega}{R_G}\right) V_C$$

The differential voltage is then converted to a single-ended voltage using the output amplifier, which also rejects any common-mode signal at the output of the input amplifiers.

Because the amplifiers can swing to either supply rail, as well as have their common-mode range extended to below the negative supply rail, the range over which the AD623 can operate is further enhanced (see Figure 20 and Figure 21).

The output voltage at Pin 6 is measured with respect to the potential at Pin 5. The impedance of the reference pin is 100 kΩ; therefore, in applications requiring V/I conversion, a small resistor between Pin 5 and Pin 6 is all that is needed.

Figure 41. Simplified Schematic

Note that the bandwidth of the in-amp decreases as gain is increased. This occurs because the internal op-amps are the standard voltage feedback design. At unity gain, the output amplifier limits the bandwidth.

FIGURE 9. A paragraph extracted from the data-sheet of AD623. Analog Devices describes it as an integrated version of the classic 3 operational amplifier instrumentation amplifier.

5. Isolation amplifiers

Isolation amplifiers are able to decouple effectively two parts of a circuit which must exchange a signal, without having a direct galvanic connection between them. Figure 10 shows a schematic view of the way they are done: the signal e_{i} is transferred through an insulation barrier. Usually the transfer is done by an optical link (optocouplers), magnetically (transformers) or capacitively. Isolation amplifiers can be required for safety reasons and protection of equipments, for example when high voltages are involved. In this case, they are effective to eliminate very high common mode voltages V_{m} between the decoupled sections. A second important reason is to avoid ground loops, yielding severe electromagnetic compatibility issues (see paragraph [**Bucci, 2017**]). This is made explicit in figure 10 by the choice of two different symbols for the reference nodes, once the isolation barrier is crossed.

Isolation amplifiers might be rated to guarantee several thousands volts of isolation. Examples include the classic ISO120 integrated isolation amplifier, whose internal structure is shown in figure 11. The isolation barrier is capacitive, so the input signal is used to modulate a carrier around 400 kHz. Note how a feedback loop is used on one side of the isolation barrier to achieve good linearity: this trick is effective when it is possible to obtain almost identical circuits on the two sides of the isolation barrier. Figure 12 contains the description of the working principle, the effect of sampling is visible in the oscillograms.

An example of a low-cost modern device proposed by Texas Instruments is the AMC1100, with once again a capacitive coupling. On the other side, the Analog Devices AD202 features a complete amplifier module, transformer-coupled, with an onboard isolated power supply converter and a cost which is aligned with the performances.

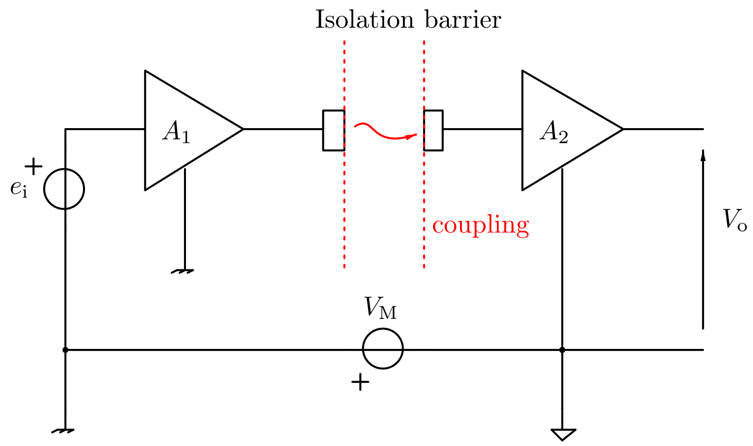


FIGURE 10. A schematic view of the principles of an isolation amplifier. The presence of an isolation barrier makes sort that the two reference nodes can be subjected to a voltage V_M without any current flowing and no risk for the signal integrity as long as V_M remains below a certain limit, specified in the datasheet.

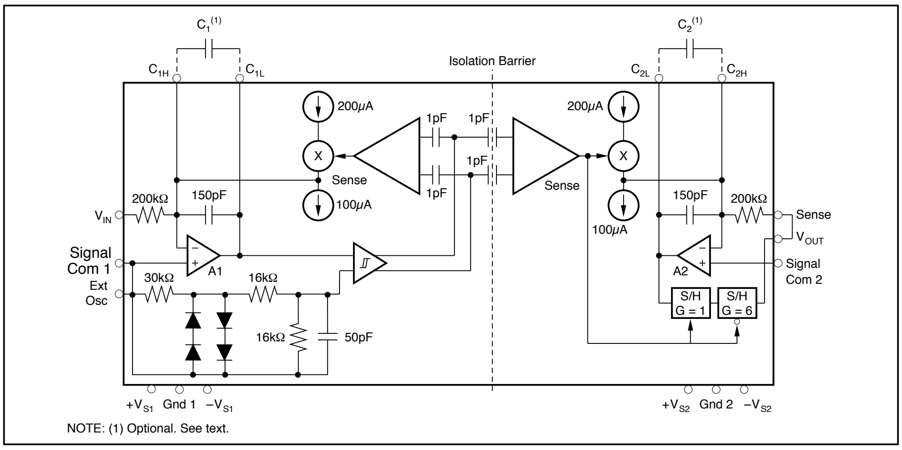
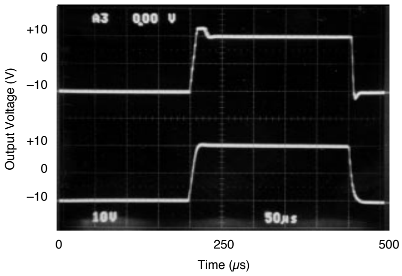


FIGURE 11. Block diagram of the internal structure of ISO120, a classic isolation amplifier from Texas Instruments.

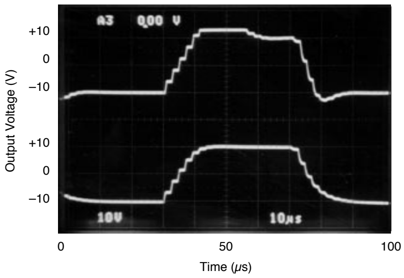
TYPICAL PERFORMANCE CURVES (CONT)

T_A = +25°C; V_{S1} = V_{S2} = ±15V; and R_L = 2kΩ, unless otherwise noted.

STEP RESPONSE



STEP RESPONSE



THEORY OF OPERATION

The ISO120 and ISO121 isolation amplifiers comprise input and output sections galvanically isolated by matched 1pF capacitors built into the ceramic barrier. The input is duty-cycle modulated and transmitted digitally across the barrier. The output section receives the modulated signal, converts it back to an analog voltage and removes the ripple component inherent in the demodulation. The input and output sections are laser-trimmed for exceptional matching of circuitry common to both input and output sections.

FREE-RUNNING MODE

An input amplifier (A1, Figure1) integrates the difference between the input current (V_{IN}/200kΩ) and a switched ±100μA current source. This current source is implemented by a switchable 200μA source and a fixed 100μA current sink. To understand the basic operation of the input section, assume that V_{IN} = 0. The integrator will ramp in one direction until the comparator threshold is exceeded. The comparator and sense amp will force the current source to switch; the resultant signal is a triangular waveform with a 50% duty cycle. If V_{IN} changes, the duty cycle of the integrator will change to keep the average DC value at the output of A1 near zero volts. This action converts the input voltage to a duty-cycle modulated triangular waveform at the output of A1 near zero volts. This action converts the input voltage to a duty-cycle modulated triangular waveform at the output of A1 with a frequency determined by the internal 150pF capacitor. The comparator generates a fast rise time square wave that is simultaneously fed back to keep A1 in charge balance and also across the barrier to a differential sense amplifier with high common-mode rejection characteristics. The sense amplifier drives a switched current source surrounding A2. The output stage balances the duty-cycle modulated current against the feedback current through the 200kΩ feedback resistor, resulting in an average value at the Sense pin equal to V_{IN}. The sample and hold amplifiers in the output feedback loop serve to remove undesired ripple voltages inherent in the demodulation process.

SYNCHRONIZED MODE

A unique feature of the ISO120 and ISO121 is the ability to synchronize the modulator to an external signal source. This capability is useful in eliminating trouble-some beat frequencies in multi-channel systems and in rejecting AC signals and their harmonics. To use this feature, external capacitors are connected at C_1 and C_2 (Figure 1) to change the free-running carrier frequency. An external signal is applied to the Ext Osc pin. This signal forces the current source to switch at the frequency of the external signal. If V_{IN} is zero, and the external source has a 50% duty cycle, operation proceeds as described above, except that the switching frequency is that of the external source. If the external signal has a duty cycle other than 50%, its average value is not zero. At start-up, the current source does not switch until the integrator establishes an output equal to the average DC value of the external signal. At this point, the external signal is able to trigger the current source, producing a triangular waveform, symmetrical about the new DC value, at the output of A1. For V_{IN} = 0, this waveform has a 50% duty cycle. As V_{IN} varies, the waveform retains its DC offset, but varies in duty cycle to maintain charge balance around A1. Operation of the demodulator is the same as outlined above.

Synchronizing to a Sine or Triangle Wave External Clock

The ideal external clock signal for the ISO120/121 is a ±4V sine wave or ±4V, 50% duty-cycle triangle wave. The *ext osc* pin of the ISO120/121 can be driven directly with a ±3V to ±5V sine or 25% to 75% duty-cycle triangle wave and the ISO amp's internal modulator/demodulator circuitry will synchronize to the signal.

Synchronizing to signals below 400kHz requires the addition of two external capacitors to the ISO120/121. Connect one capacitor in parallel with the internal modulator capacitor and connect the other capacitor in parallel with the internal demodulator capacitor as shown in Figure 1.

FIGURE 12. Another extract of ISO120 data-sheet. Here is Ti's description of how the device works.

6. Conclusion

In this document, we saw rapidly the main characteristics of operational amplifiers. Then, we gave the desirable characteristics of instrumentation amplifiers and described three variants, often employed in practical situations. We finished our discussion by presenting isolation amplifiers.

Bibliography

[Bucci, 2021] D. Bucci, *Capteurs et conditionneurs*, polycopyé por le tronc commun PI, Instrumentation Electronique, 2021/2022.

[Asch, 2003] G. Asch et collaborateurs, *Acquisition de donnes, du capteur l'ordinateur*, 2ème dition, Dunod, Paris 2003.

[Franco, 2002] S. Franco, *Design with operational amplifiers and analog integrated circuits*, 2nd dition, McGrawHill, 2002.

[Bucci, 2017] D. Bucci, Analog Electronics for Measuring Systems, ISTE/WILEY 1st ed., 2017.