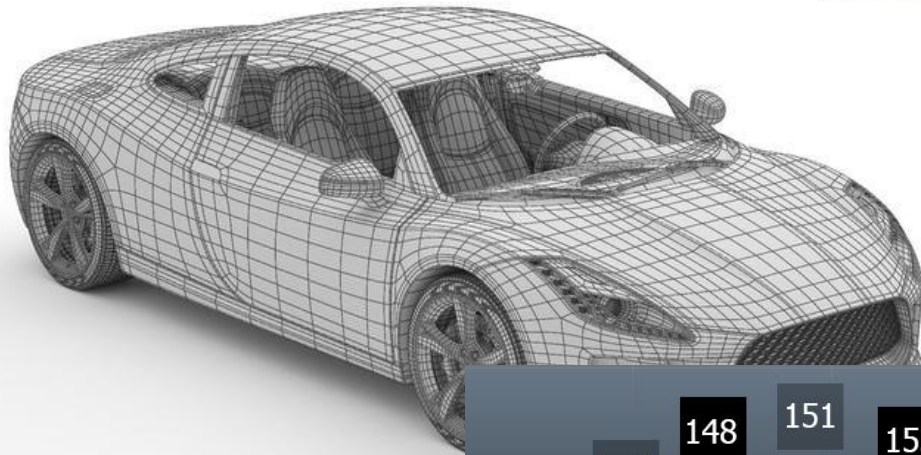


3D GRAPHICS



design



animate



render

Computer Graphics

- 3D animation movies



Computer Graphics

- Special effects



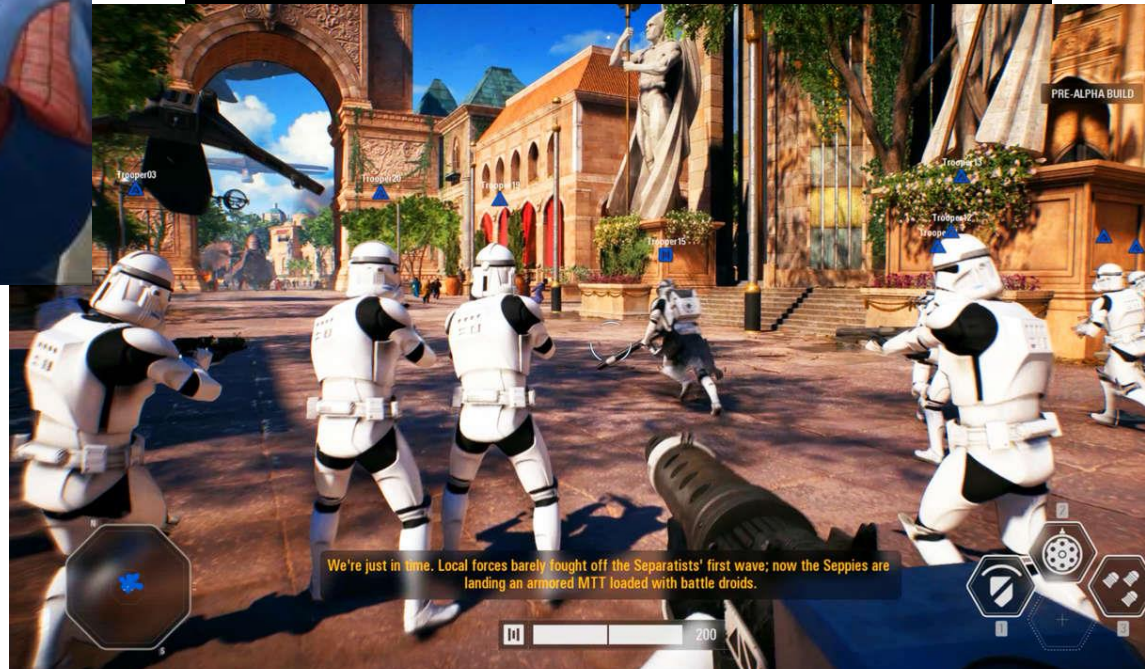
Computer Graphics

- Advertising



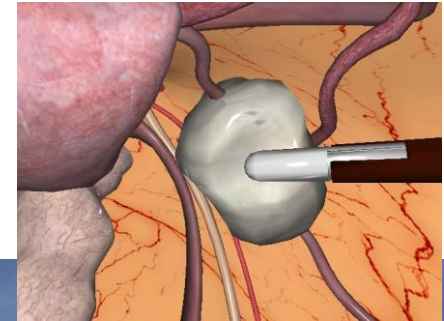
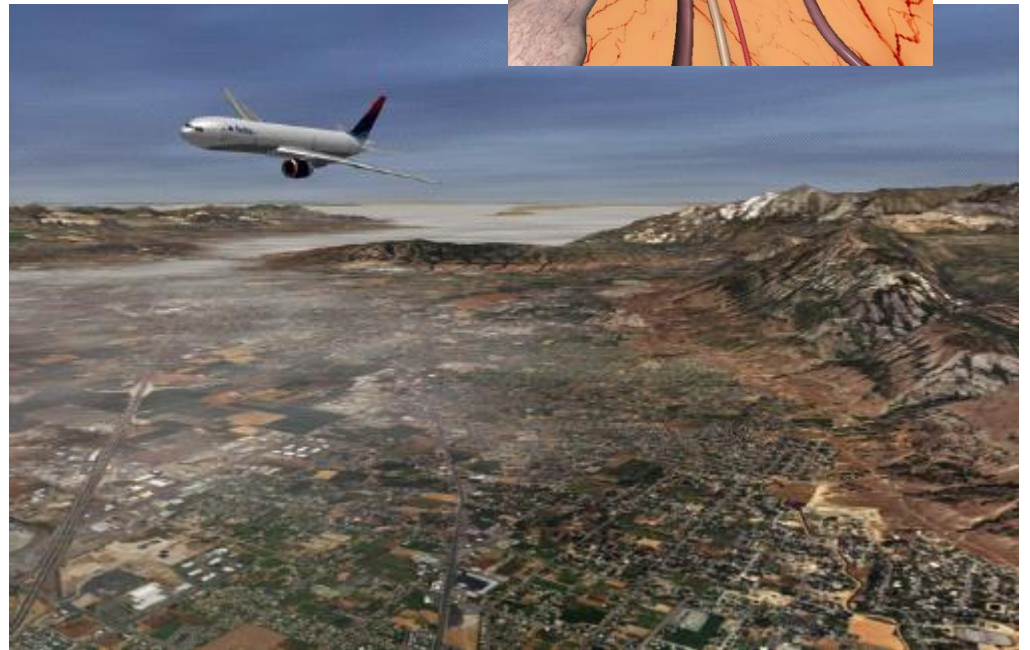
Computer Graphics

- Games



Computer Graphics

- Simulations & serious games



Computer Graphics

- Computer Aided Design (CAD)



Computer Graphics

- Architecture



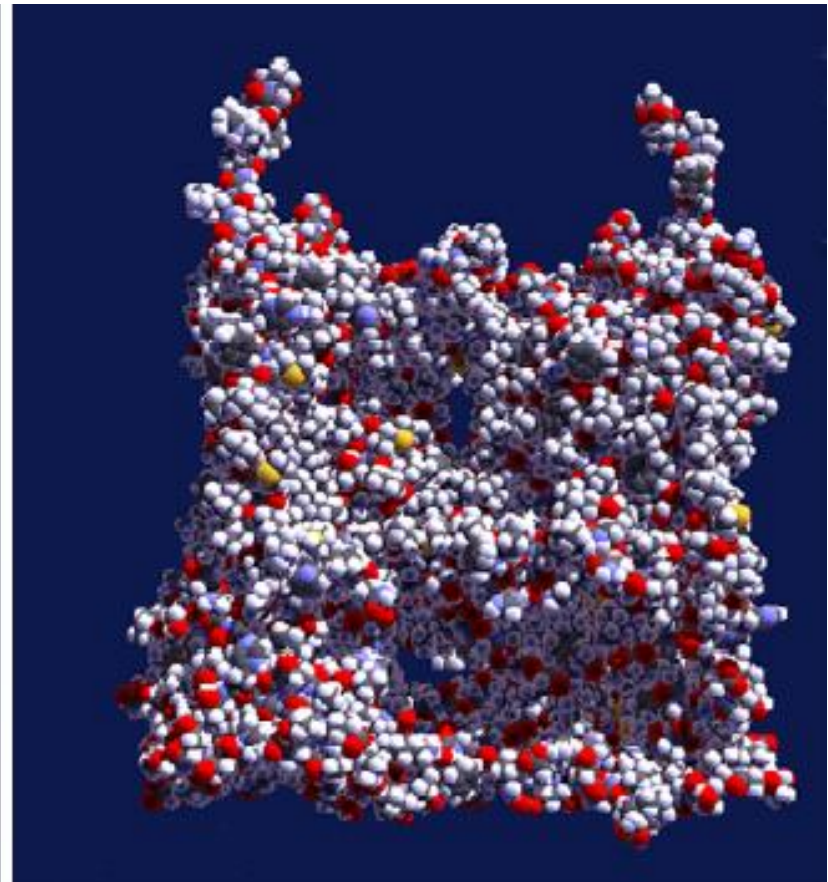
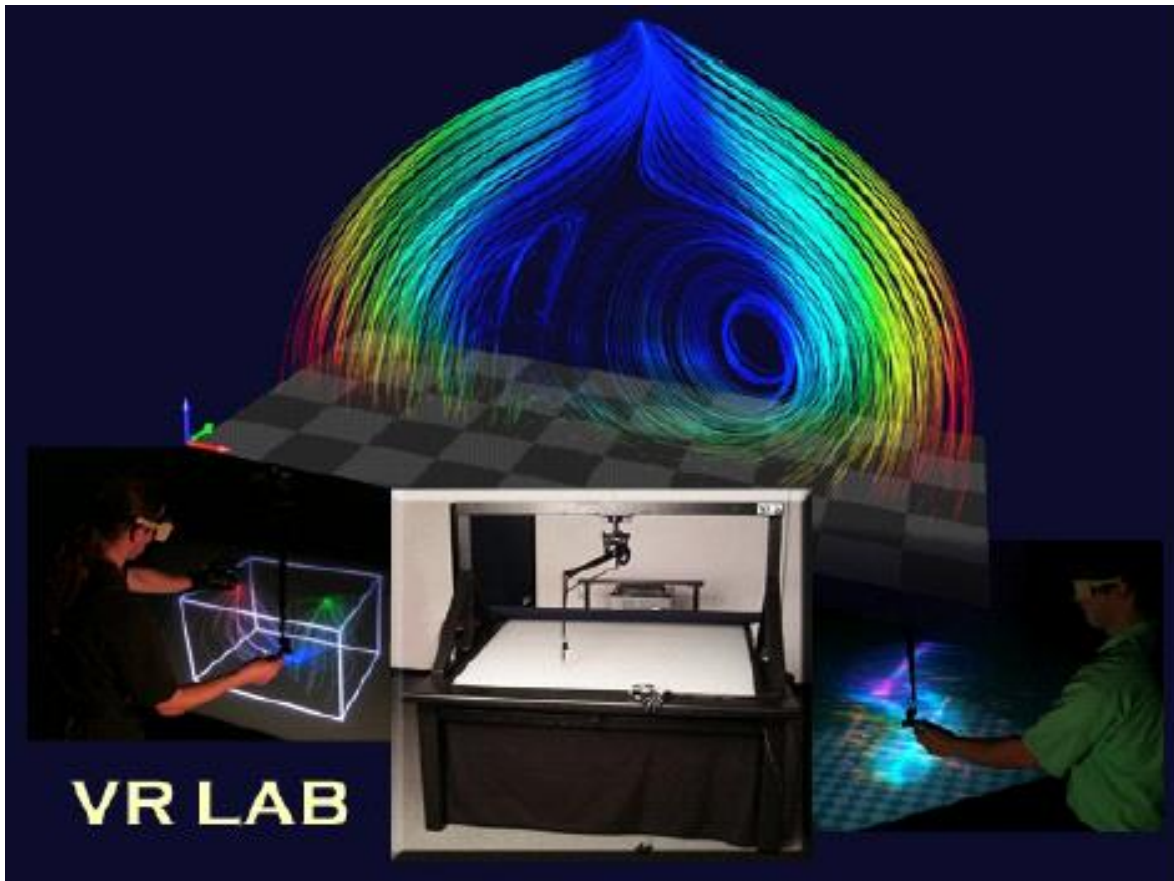
Computer Graphics

- Virtual/augmented reality



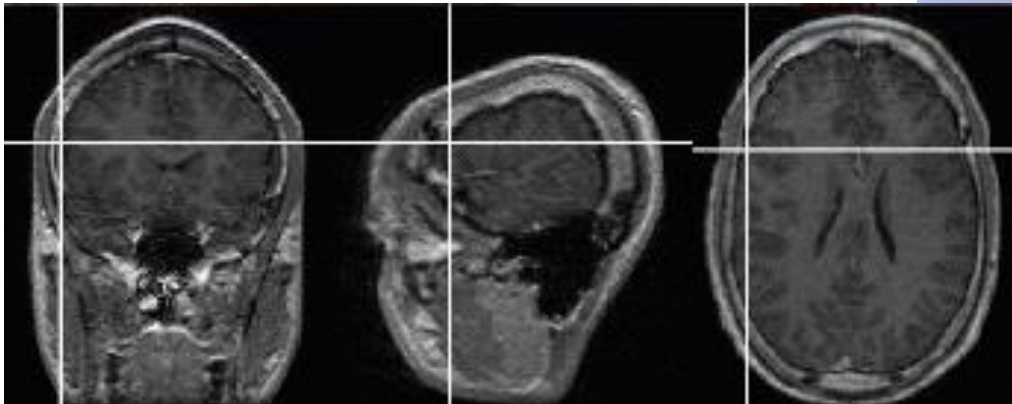
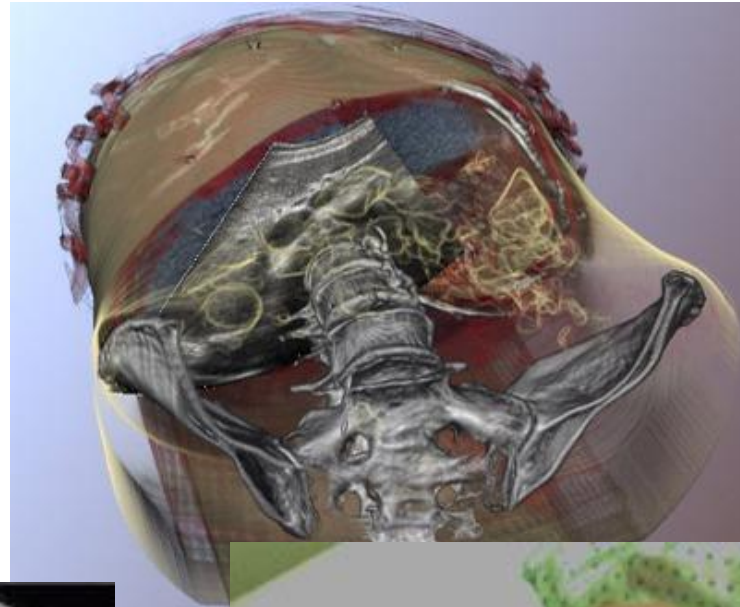
Computer Graphics

- Visualization

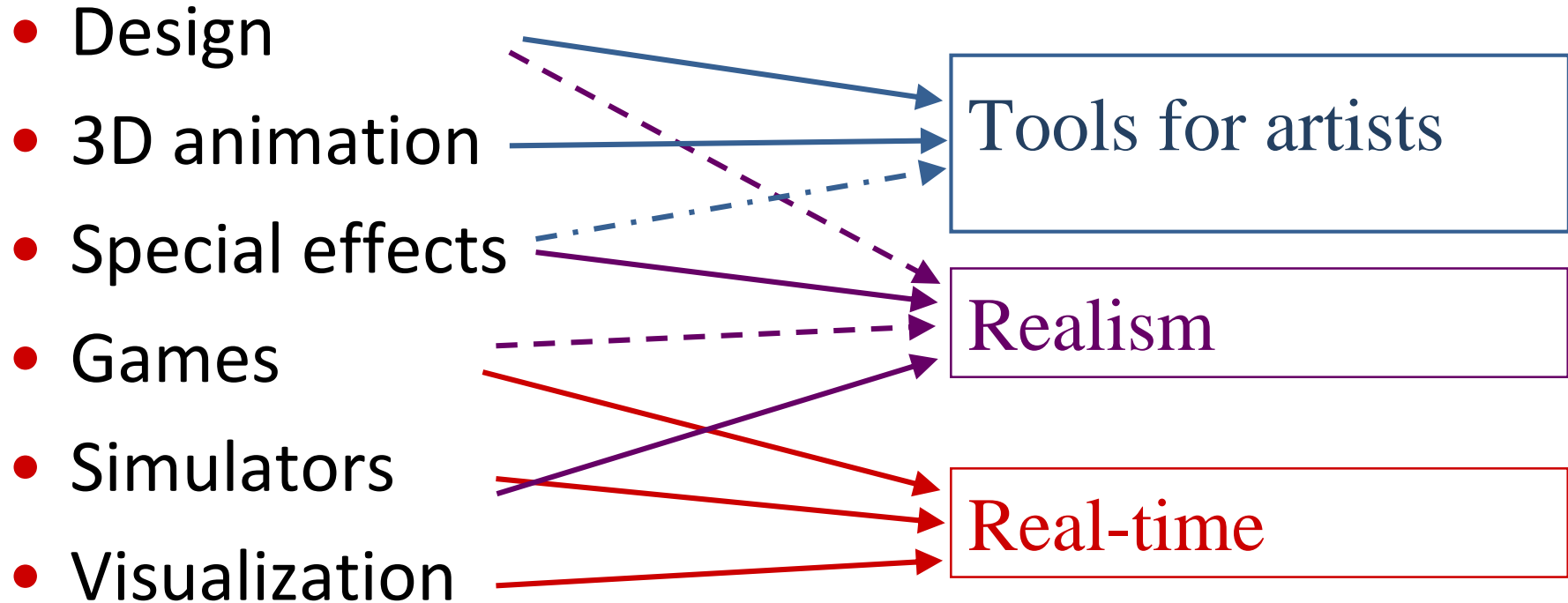


Computer Graphics

- Medical imaging



Computer Graphics



What you will learn

1. Overview of Computer Graphics (for engineering & research)
 - Modeling : create 3D geometry
 - Animation : move & deform
 - Rendering : 3D scene → image
2. How the basic techniques work
3. Practice with OpenGL (+Python)
4. Case studies : Practical problems
 - How to choose & combine existing techniques (TD)



*What you will **not** learn*

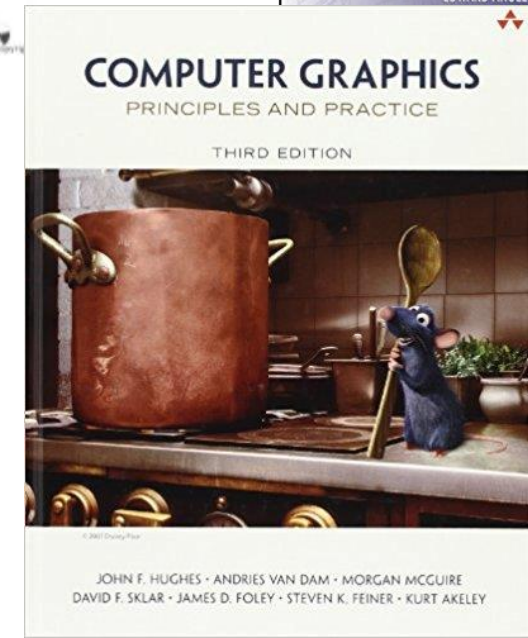
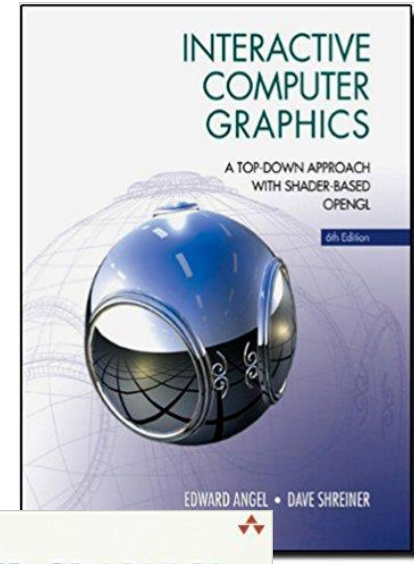
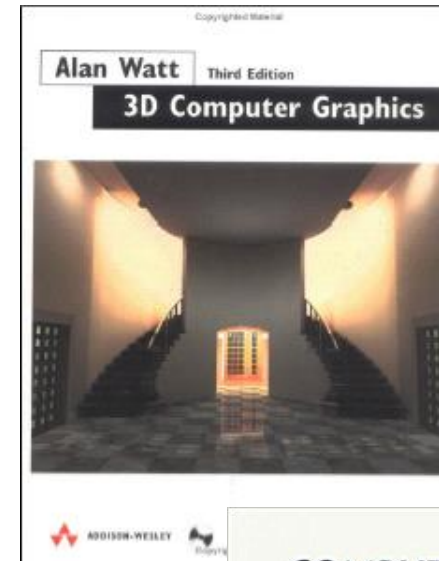
- Mathematical bases (algebra, geometry!)
- Advanced CG techniques in detail
- Advanced Programming the Graphics Hardware (GPU)
- Artistic skills or Game design
- Use of existing software
(CAD-CAM, 3D Studio Max, Maya, Photoshop, etc)

Text books

No book required

References

- *3D Computer Graphics* (Watt, 2000)
- *Interactive Computer Graphics* (Angel & Shreiner, 20).
- *Computer Graphics: Principles and Practices* (Hughes et al. 2013)



Organization & overview

1.5h CTD (course & exercises) + 1.5h TP (lab)

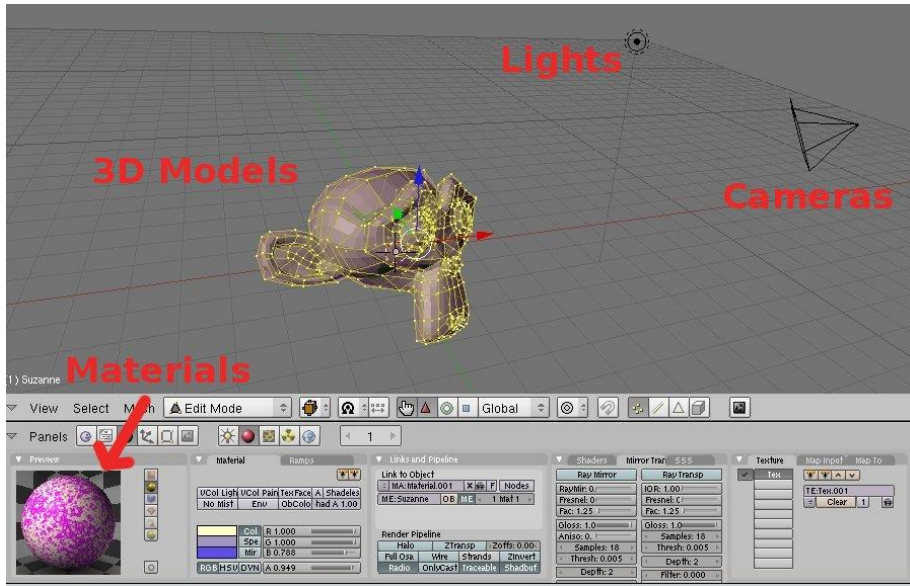
- *Part 1: Basic techniques*
 1. Modeling: geometric representations, hierarchical modeling
 2. Rendering: illumination, shading, textures
 3. Animation: Keyframing, skinning, collisions
- *Part 2: Introduction to advanced methods*

3 advanced courses on modeling/rendering/animation

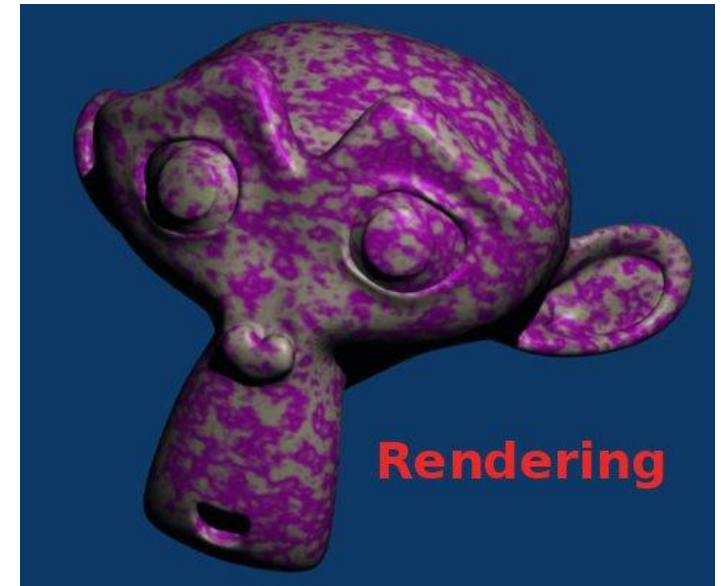
Evaluation: [0,5 Exam + 0,5 OpenGL project]

1: graphics (projective) pipeline

Inputs



Output



Implemented in the Graphics Hardware (real time)
Used by OpenGL

1: graphics pipeline

Required: transformations using 4x4 matrices

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$V' = M V$$

1: graphics pipeline

Required: transformations using 4x4 matrices

- “Homogeneous coordinates”
- Needed for projective transformations
- Cartesian to projective coordinates: **w=1**
- Projective to Cartesian coordinates: **divide by w**

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$V' = M V$$

1: graphics pipeline

sub-case: affine transformations

- Start with cartesian coordinates: $w=1$
- Keep the last line to 1
- The last matrix column enables to express translations!

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$V' = M V$$

Typical affine transformations

Translation

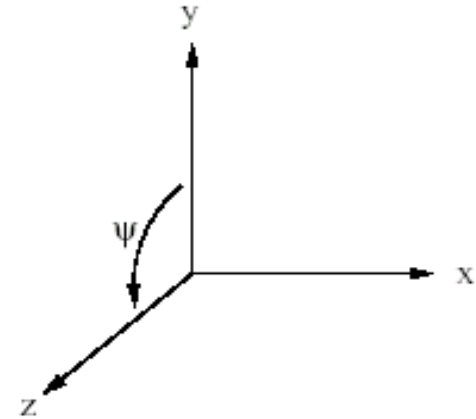
$$T = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scale

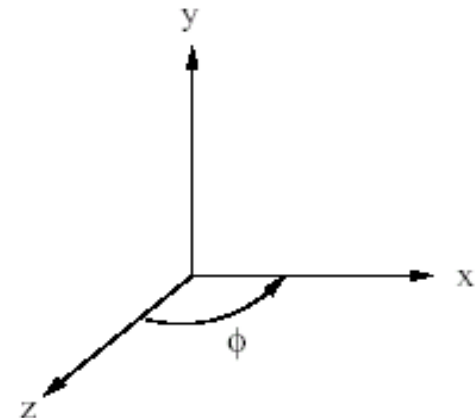
$$T = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Rotations (Euler angles)

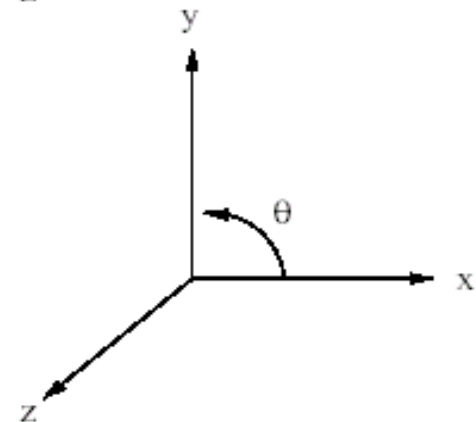
$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \psi & -\sin \psi & 0 \\ 0 & \sin \psi & \cos \psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$R_y = \begin{bmatrix} \cos \phi & 0 & \sin \phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \phi & 0 & \cos \phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



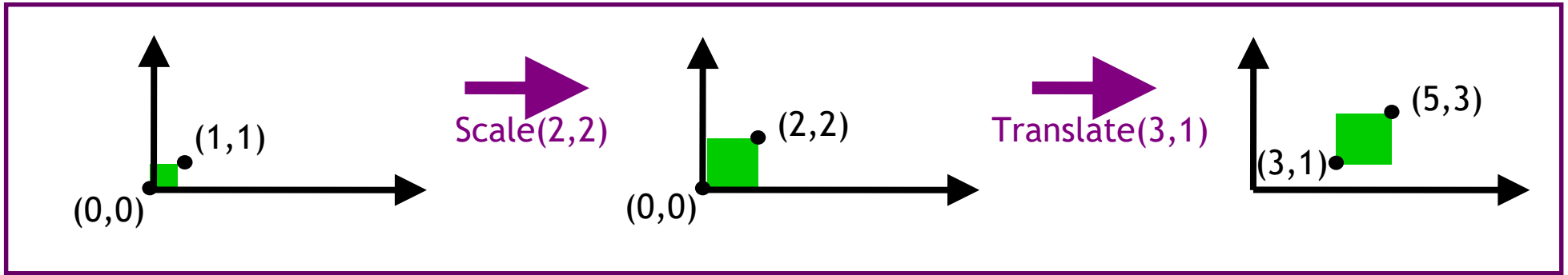
$$R_z = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$R = R_z \cdot R_y \cdot R_x,$$

2D example

Composition of scale and translate

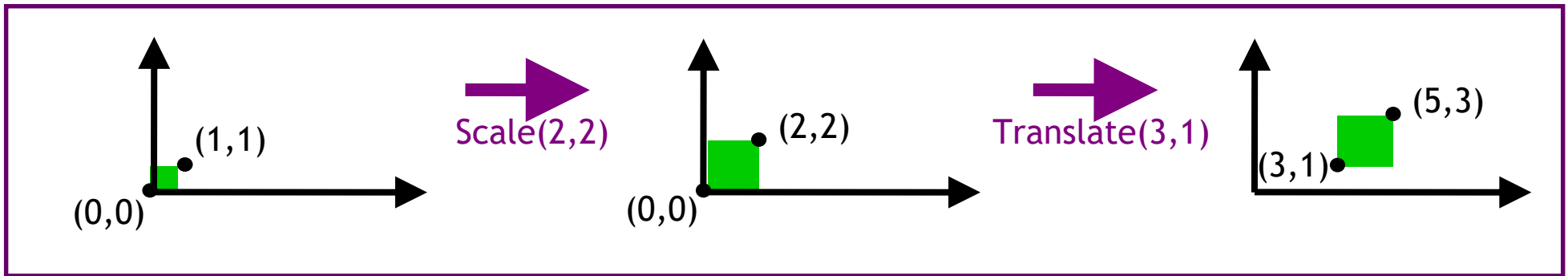


Multiplication of matrices : $p' = T (S p) = TS p$

$$TS = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

TD part

1. *Is the order of transformation important?*



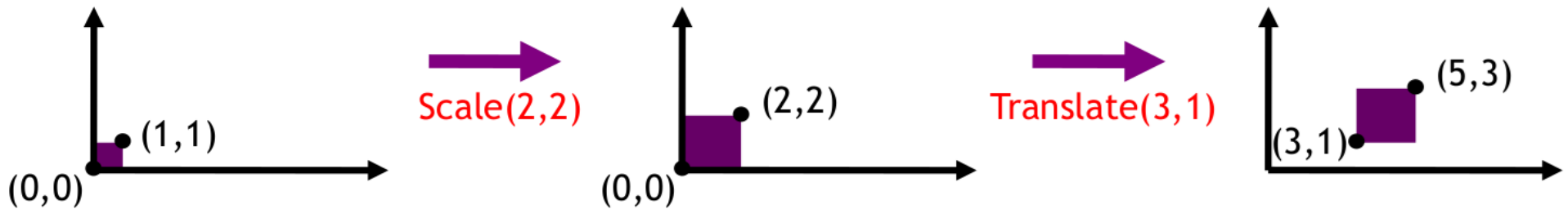
Exercise : $(T S) p \stackrel{=?}{=} (S T) p$

$$ST = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} ? \\ ? \\ ? \end{pmatrix}$$

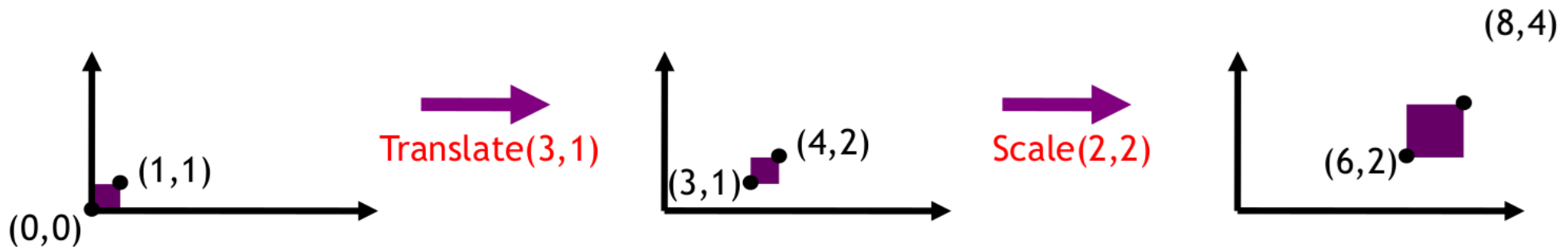
Draw the transformed polygon(s) using ST and TS

1. Solution: transformations not commutative!!

Scale then translate : $p' = T (S p) = TS p$

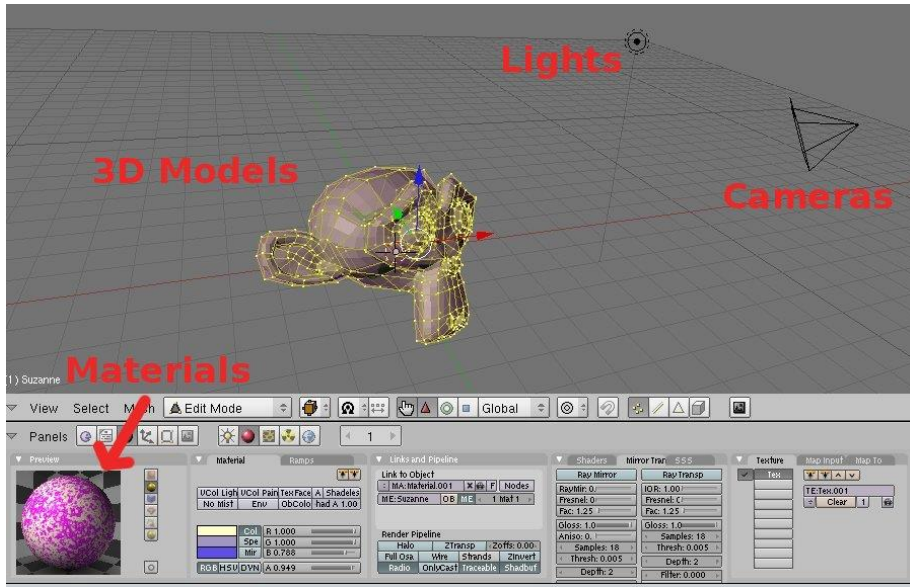


Translate, then scale : $p' = S (T p) = ST p$

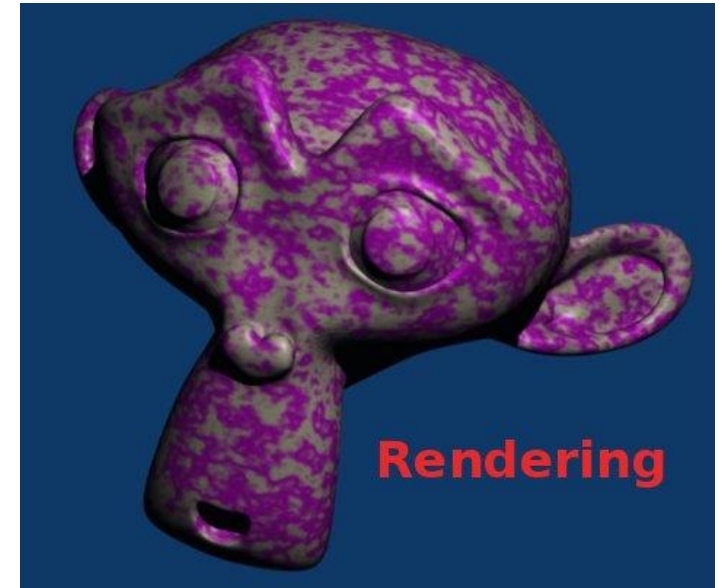


1: graphics pipeline

Inputs



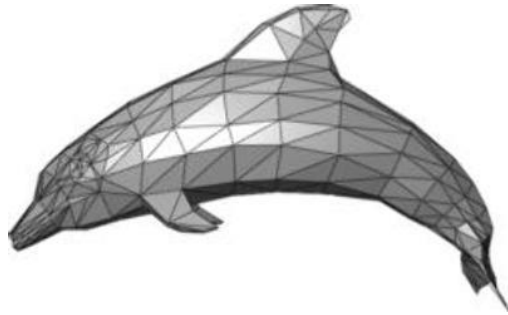
Output



Ok, back to our main question!

1: graphics pipeline

Lets consider that we already have the input data (ignore materials and lights for now)



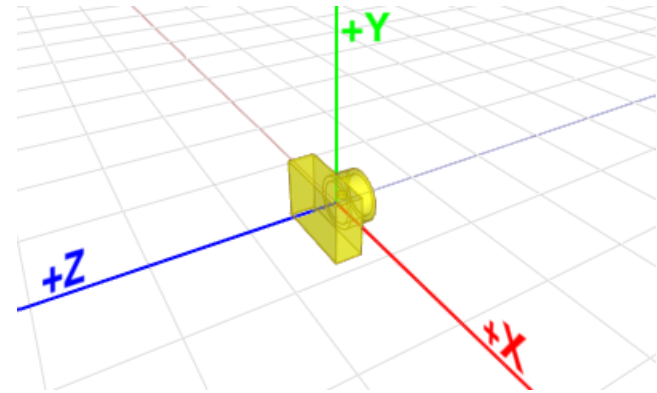
Mesh, composed of triangle faces (v_1, v_2, v_3)
Each vertex contains 3 coords (x, y, z)
defined in the local/model frame

$v_1 = (x_1, y_1, z_1)$

$v_2 = (x_2, y_2, z_2)$

...

(more in next lecture)

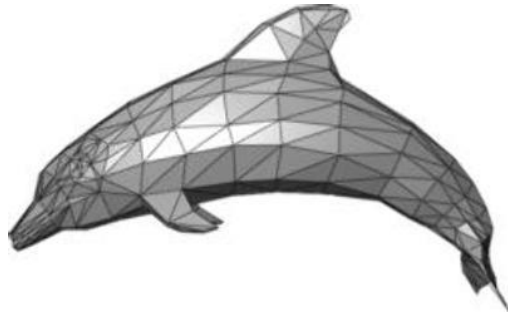


Camera, composed of 4x4 matrices

(more in a few minutes)

1: graphics pipeline

Lets consider that we already have the input data (ignore materials and lights for now)



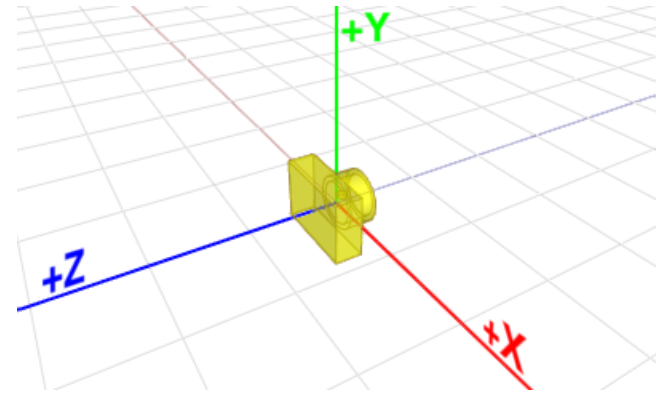
Mesh, composed of triangle faces (v_1, v_2, v_3)
Each vertex contains 3 coords (x, y, z)
defined in the local/model frame

$v_1 = (x_1, y_1, z_1)$

$v_2 = (x_2, y_2, z_2)$

...

(more in next lecture)



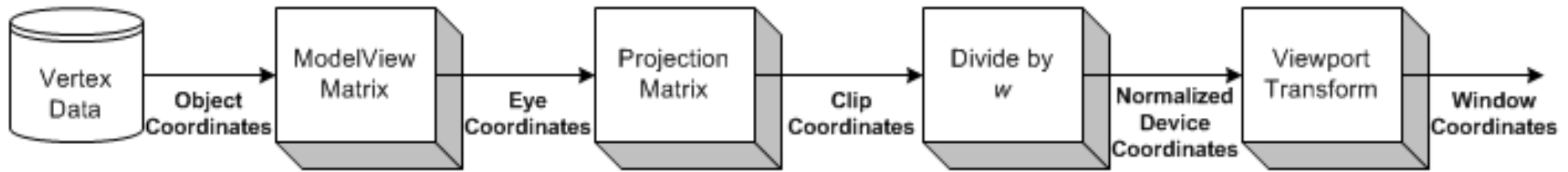
Camera, composed of 4x4 matrices

(more in a few minutes)

Creating an image from these data can be done in 4 steps!

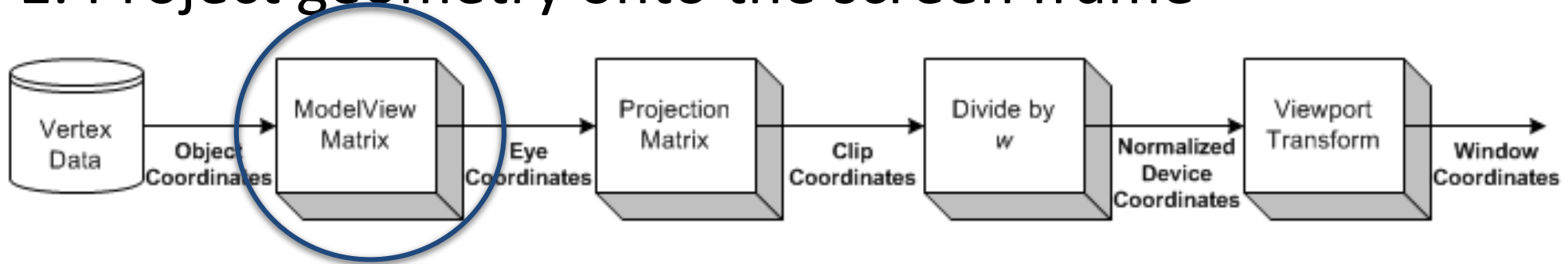
1: graphics pipeline

1. Project geometry onto the screen frame

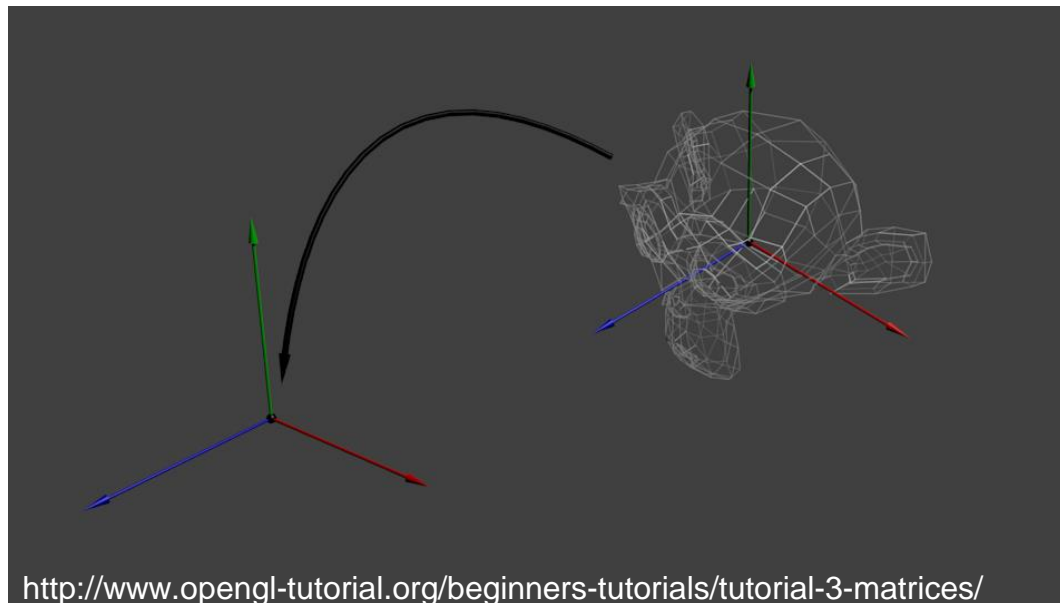


1: graphics pipeline

1. Project geometry onto the screen frame

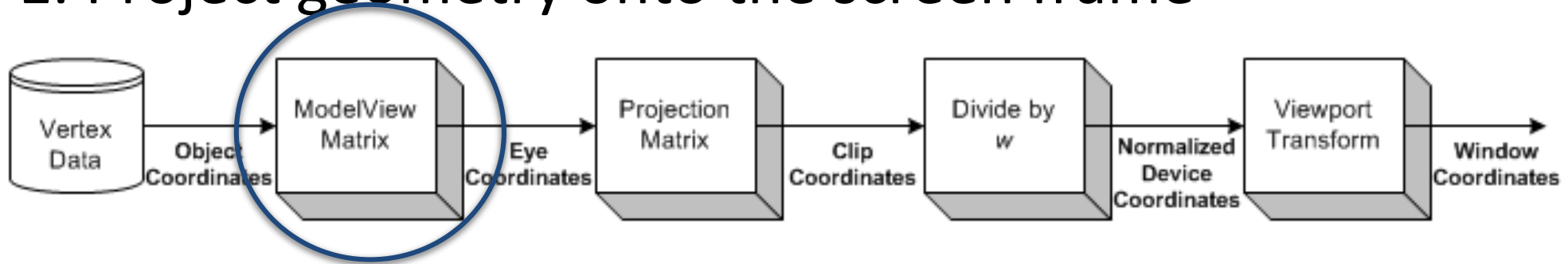


- From model to world space (4x4 matrix)

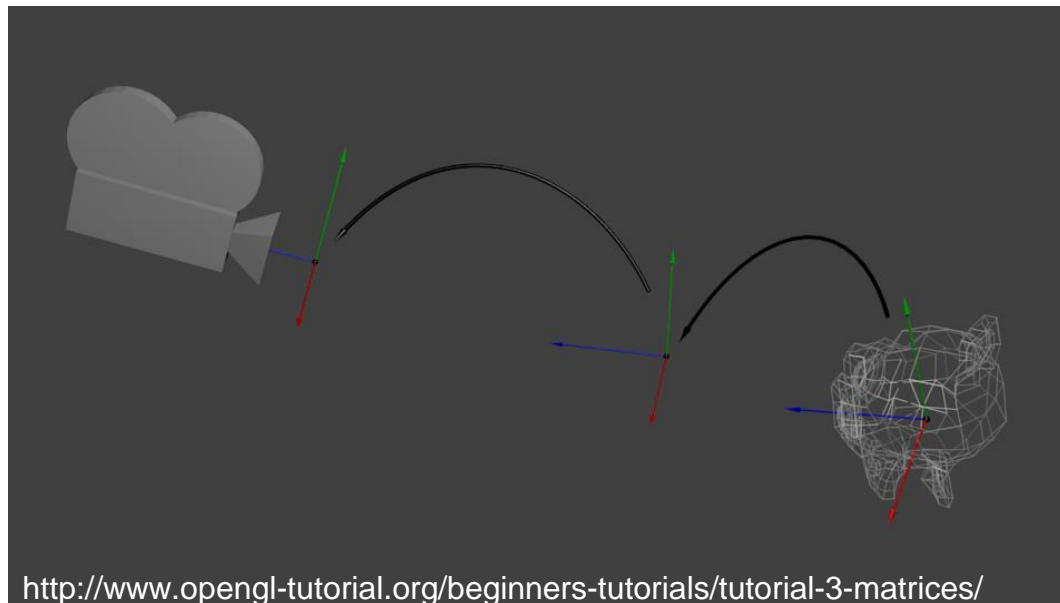


1: graphics pipeline

1. Project geometry onto the screen frame

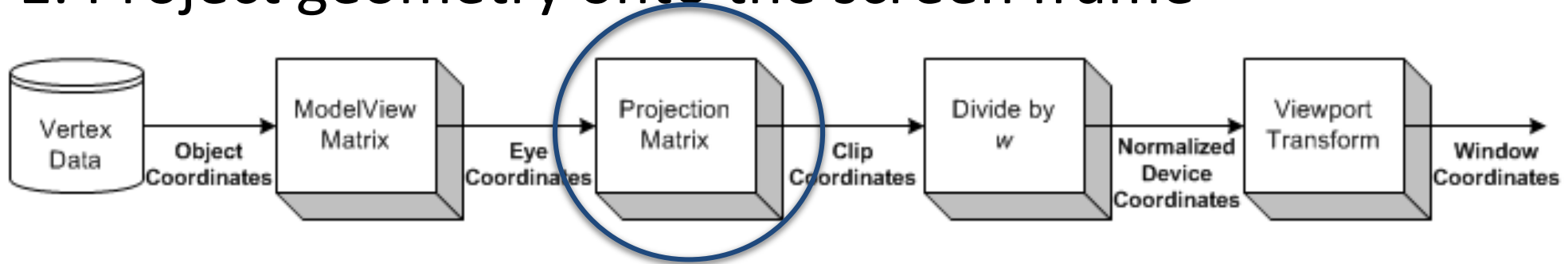


- From model to world space (4x4 matrix)
- From world to camera space (4x4 matrix)

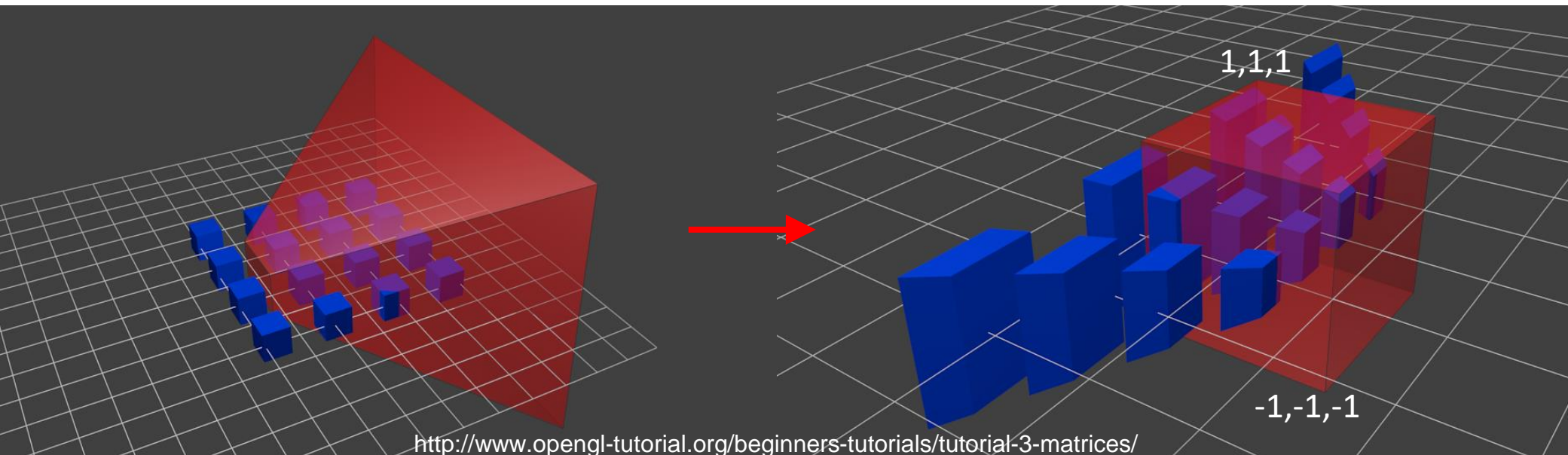


1: graphics pipeline

1. Project geometry onto the screen frame

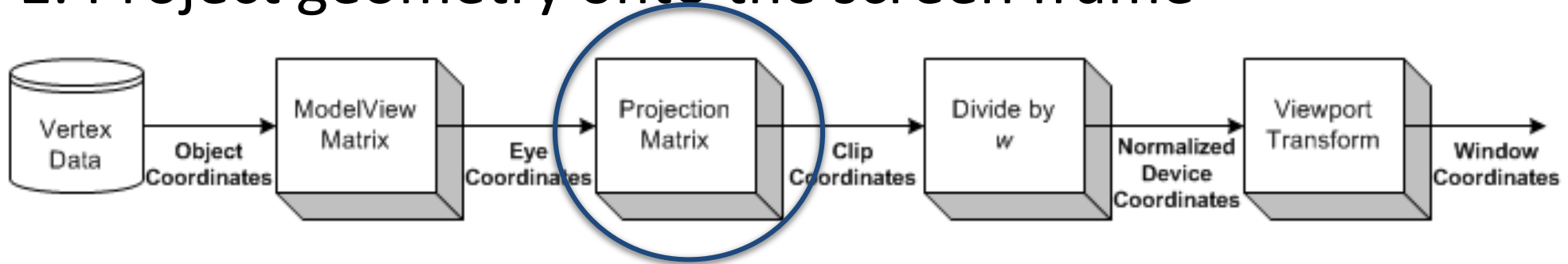


- From model to world space (4x4 matrix)
- From world to camera space (4x4 matrix)
- From camera to “screen” space (4x4 matrix)

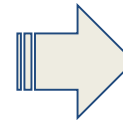


1: graphics pipeline

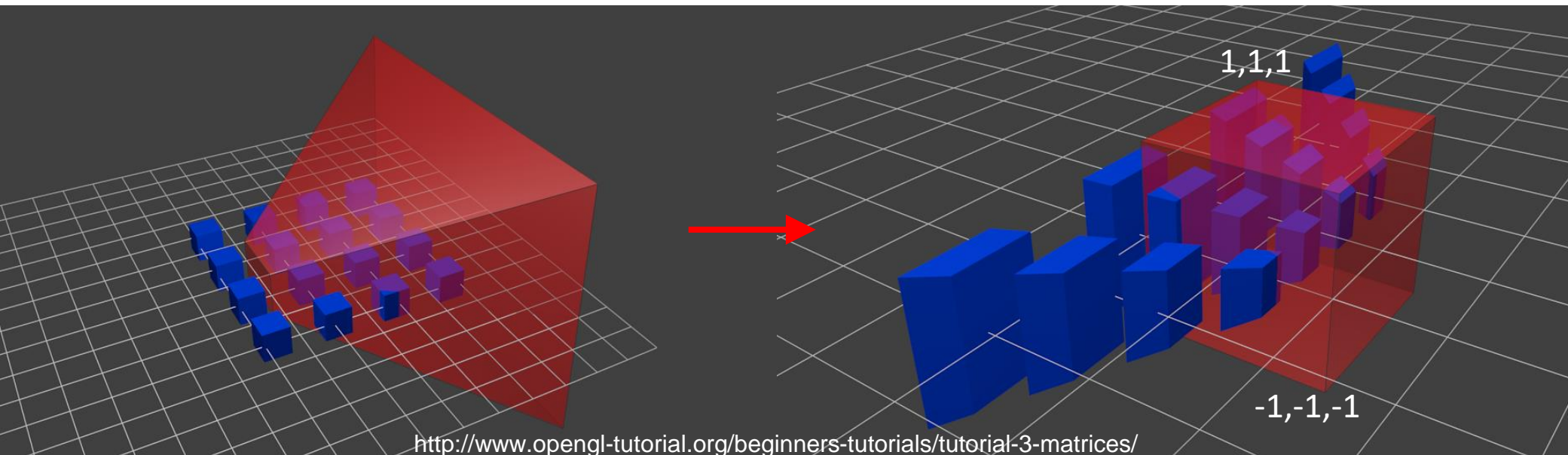
1. Project geometry onto the screen frame



- From model to world space (4x4 matrix)
- From world to camera space (4x4 matrix)
- From camera to “screen” space (4x4 matrix)

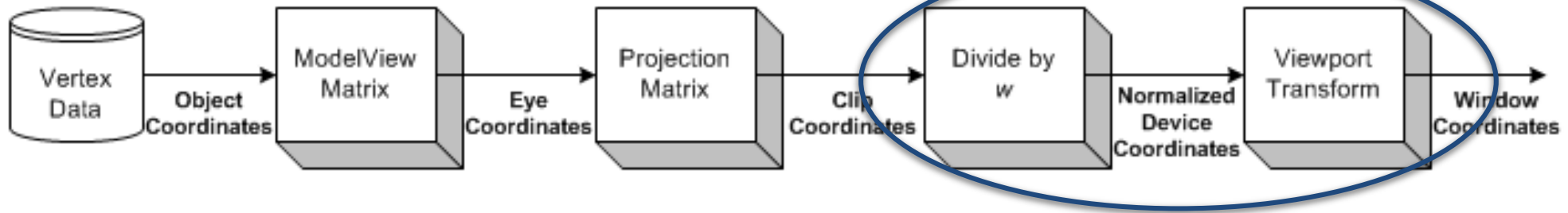


can be concatenated!



1: graphics pipeline

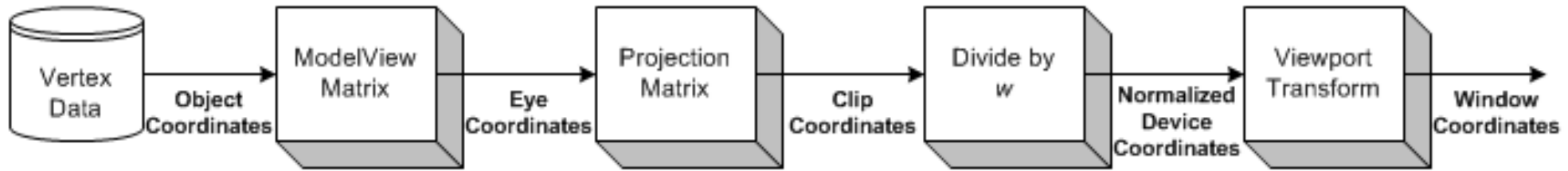
1. Project geometry onto the screen frame



- Back to cartesian coordinates...
- ... and to screen coordinates

1: graphics pipeline

1. Project geometry onto the screen frame



Questions:

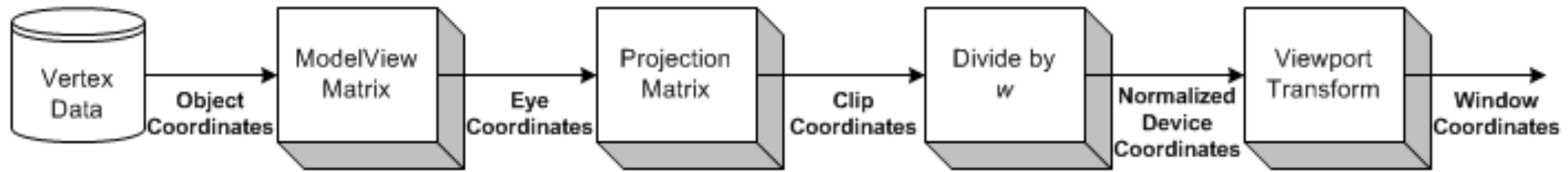
- How to (intuitively) define a camera matrix?

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{pmatrix} \begin{array}{|c|} \hline a \\ \hline e \\ \hline i \\ \hline 0 \end{array} & \begin{array}{|c|} \hline b \\ \hline f \\ \hline j \\ \hline 0 \end{array} & \begin{array}{|c|} \hline c \\ \hline g \\ \hline k \\ \hline 0 \end{array} & \begin{array}{|c|} \hline d \\ \hline h \\ \hline l \\ \hline 1 \end{array} \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

The matrix is partitioned into four columns labeled r , u , v , and t at the top. The first three columns (blue, orange, and green borders) have their bottom elements (0, 0, 0) in red. The fourth column (purple border) has its bottom element (1) in red.

1: graphics pipeline

1. Project geometry onto the screen frame



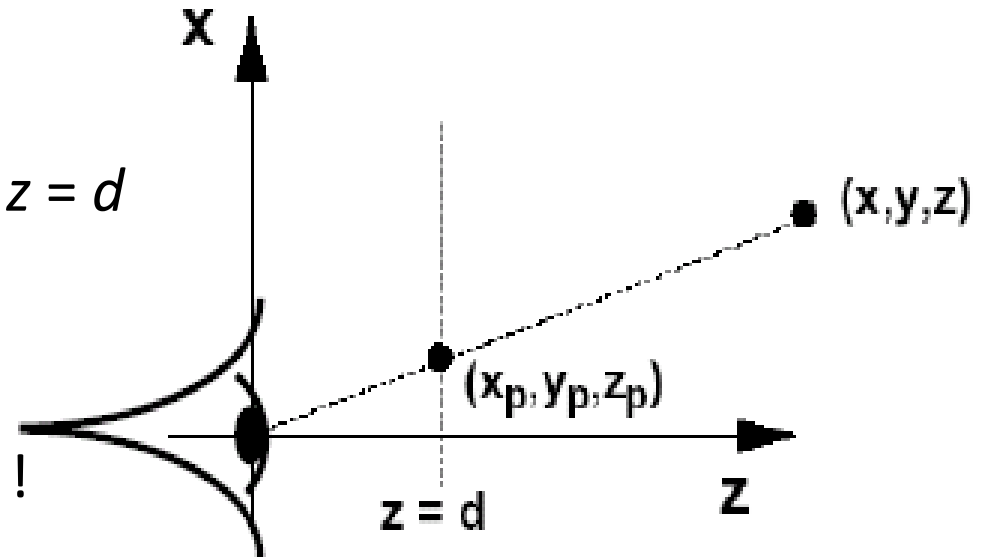
Questions:

- Finding a projection matrix

Project all points to the image plane $z = d$

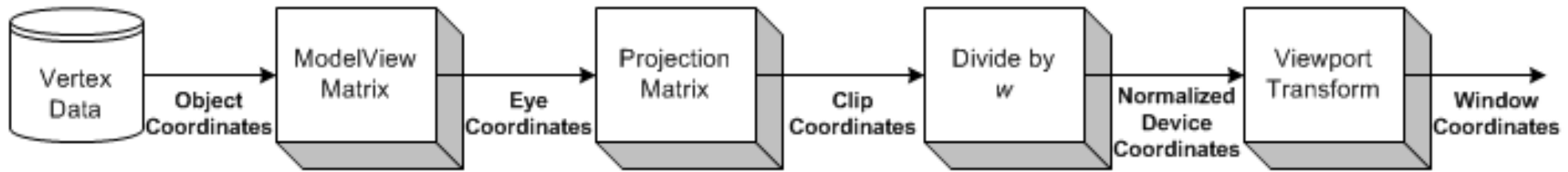
1. Compute x_p and y_p
2. Find the 4x4 matrix M needed

M should be independent from x, y, z !



1: graphics pipeline

1. Project geometry onto the screen frame



Thales theorem

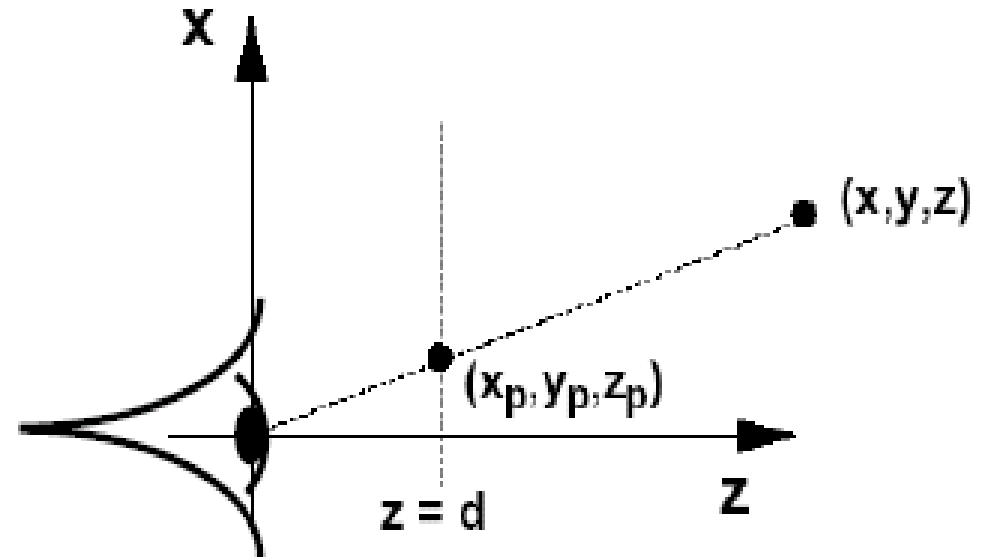
$$x_p/z_p = x/z, \text{ with } z_p=d$$

$$y_p/z_p = y/z, \text{ with } z_p=d$$

$$x_p = \frac{d \cdot x}{z} = \frac{x}{z/d}$$

$$y_p = \frac{d \cdot y}{z} = \frac{y}{z/d}$$

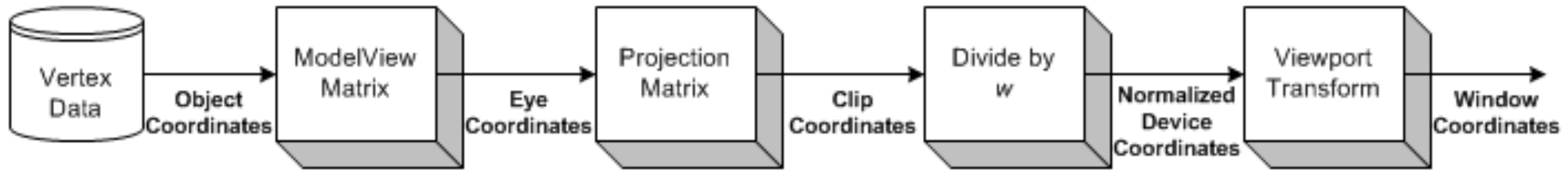
$$z_p = d$$



→ Find a transform M that divides by z/d

1: graphics pipeline

1. Project geometry onto the screen frame



Solution:

- A transform that divides all coords by $z/d \rightarrow$ set w to z/d

homogenize

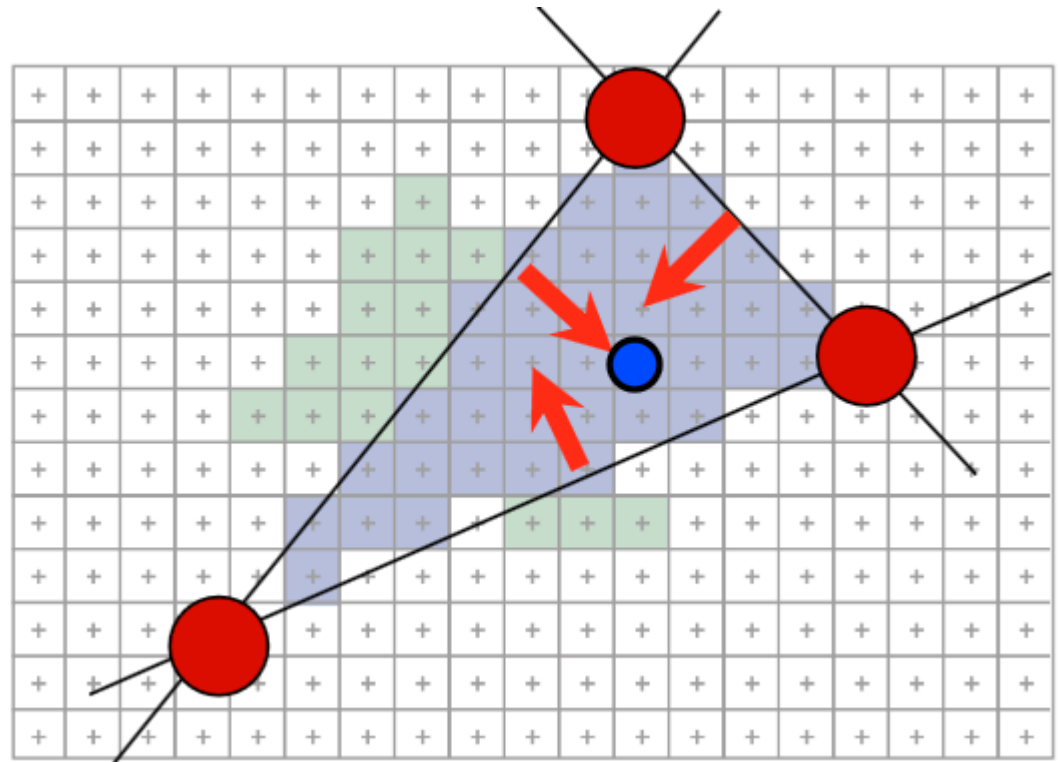
$$\text{New 3D point} = \begin{pmatrix} x * d / z \\ y * d / z \\ d \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \\ z / d \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Projective transform

1: graphics pipeline

1. Project geometry onto the screen frame
2. Rasterize triangles

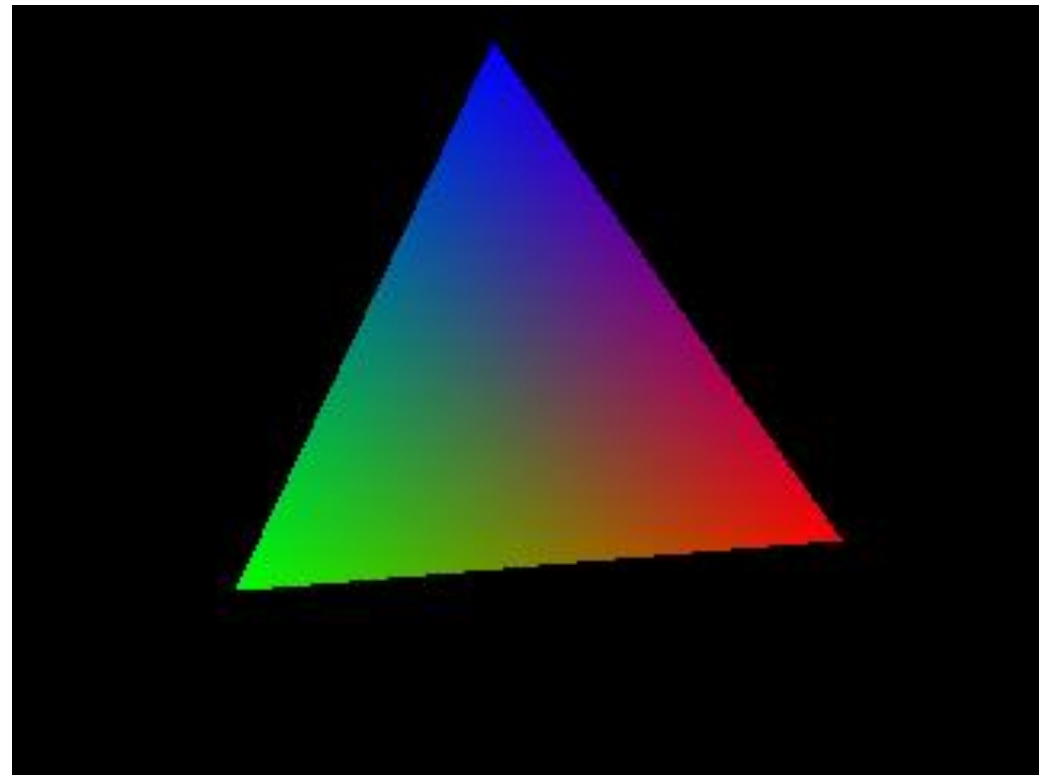
- For each pixel
 - test 3 edge equations
 - if all pass, draw
- Interpolate vertex data
 - using barycentric coords
 - positions/normals/colors/...



1: graphics pipeline

1. Project geometry onto the screen frame
2. Rasterize triangles

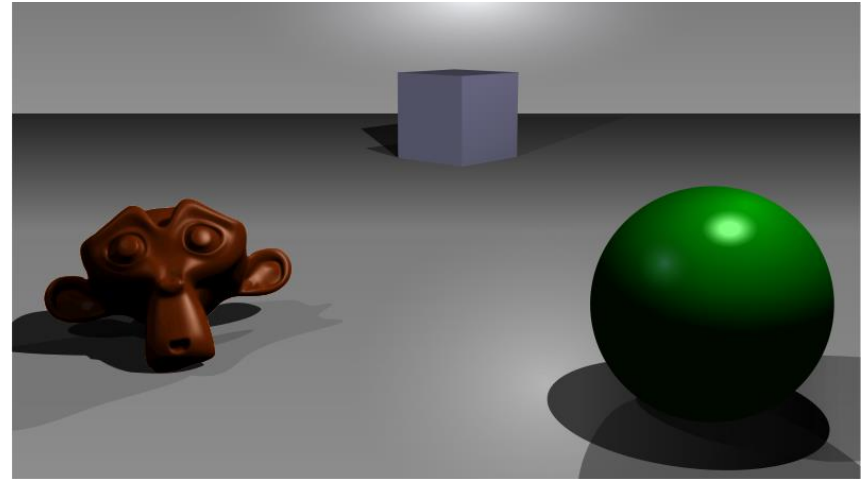
- For each pixel
 - test 3 edge equations
 - if all pass, draw
- Interpolate vertex data
 - using barycentric coords
 - positions/normals/colors/...



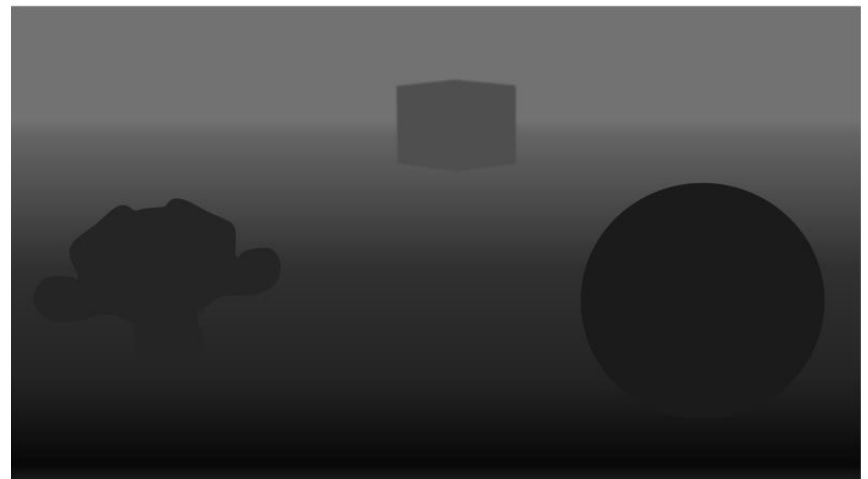
1: *graphics pipeline*

1. Project geometry onto the screen frame
2. Rasterize triangles
3. Visibility test

- For each pixel
 - Store min distance to camera
 - in a “Z-Buffer”
- if $\text{new_z} < \text{Z-Buffer}[x,y]$
 - $\text{Z-Buffer}[x,y] = \text{new_z}$
 - $\text{Framebuffer}[x,y] = \text{computePixelColor}()$



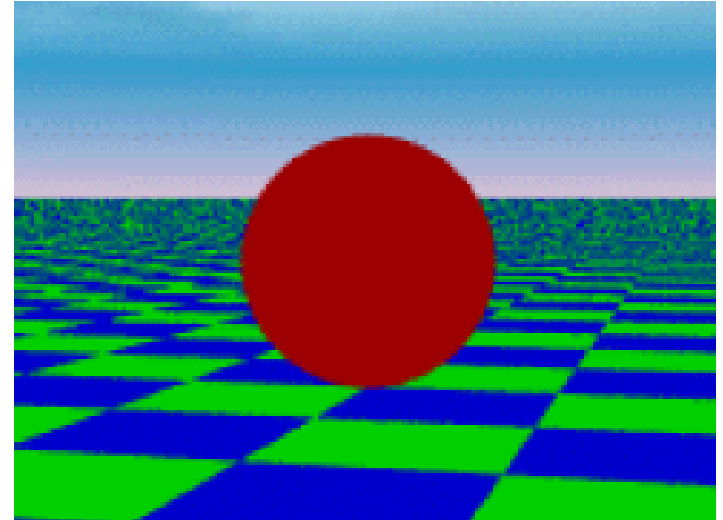
A simple three-dimensional scene



Z-buffer representation

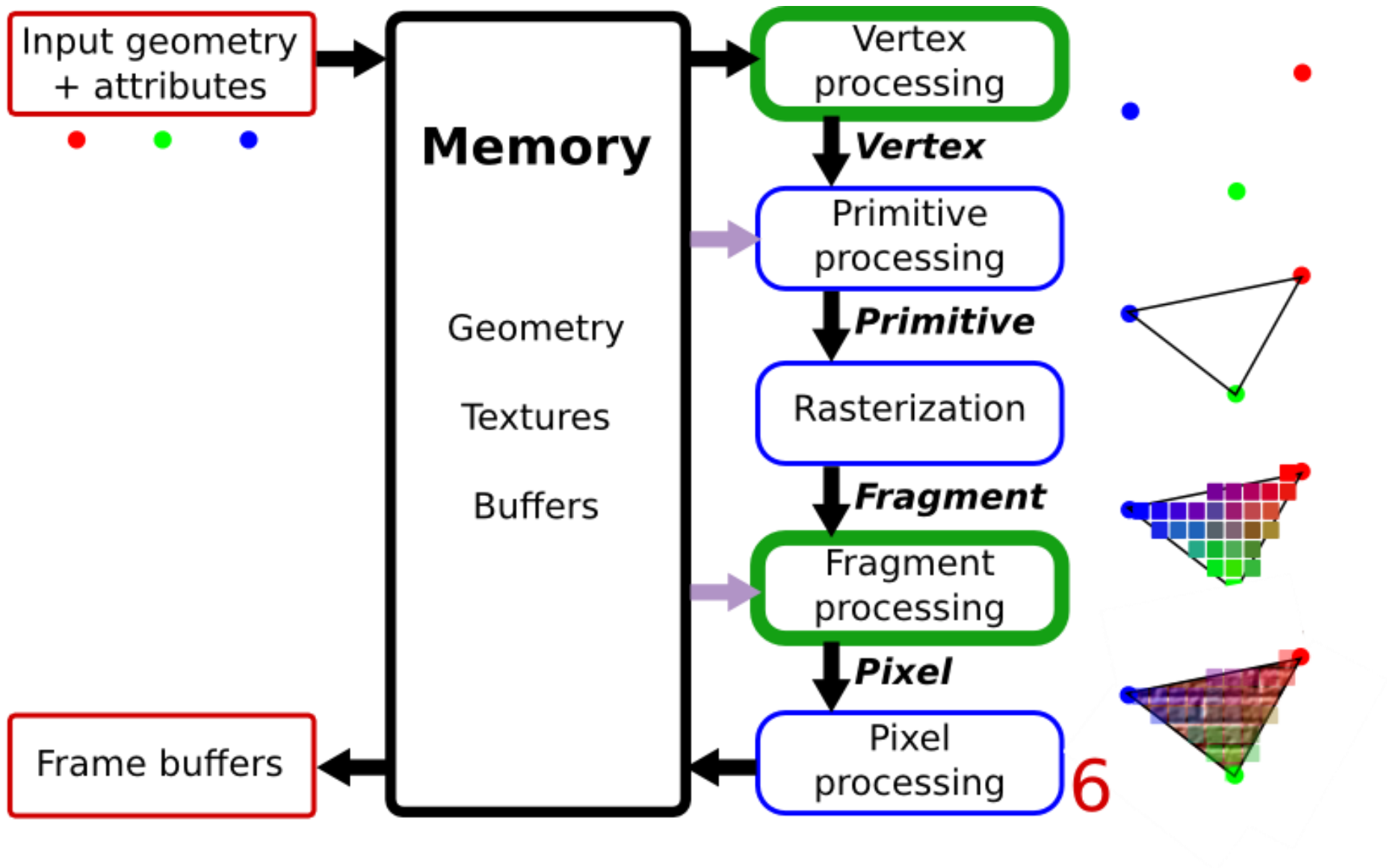
1: graphics pipeline

1. Project geometry onto the screen frame
2. Rasterize triangles
3. Visibility test
4. Compute pixel color

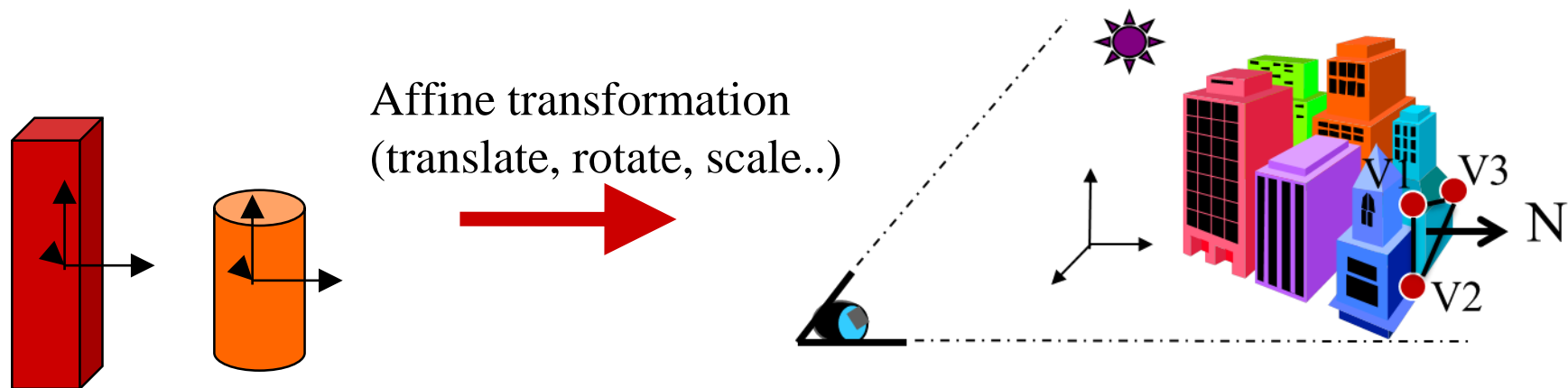


- Require more than a simple uniform color
 - This is where we will use material and lighting properties
 - to be continued... in next lectures

OpenGL pipeline



2. Transforming normal vectors?



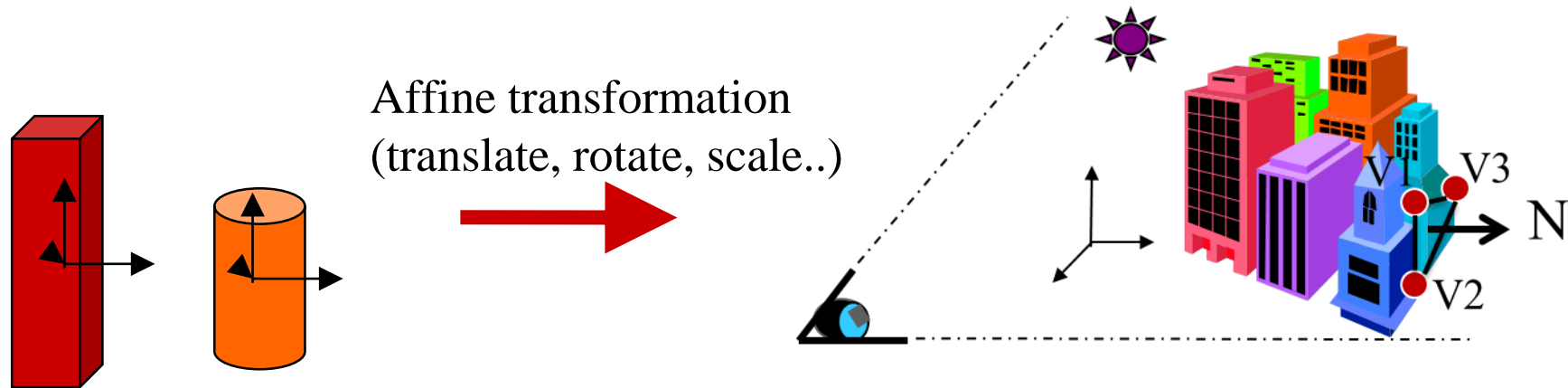
Exercise: How should we transform normal vectors?

Advice : think of the difference between

- affine transformation of points
- linear transformations of vectors

Bonus

2. Transforming normal vectors?



Apply the same transform to vectors **except translations!**

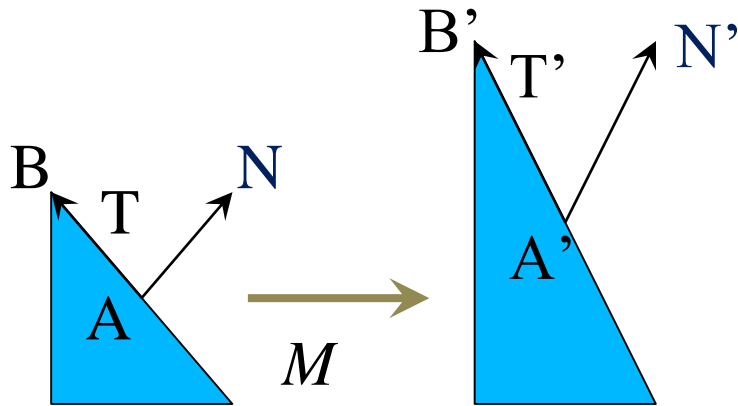
Vectorial transform

$$\begin{bmatrix} x' \\ y' \\ z' \\ 0 \end{bmatrix} = \begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{bmatrix} x \\ y \\ z \\ 0 \end{bmatrix}$$

Same 4x4 matrix
Set $w=0$ for vectors

2. Transforming normal vectors?

PB: the normal to a triangle does not remain a normal after scaling!



It works for tangent vectors:

$$T = B - A \quad T' = MT = MB - MA = B' - A'$$

How should we transform normals?

Defined by: $N \cdot T = 0$

- We are looking for G such that $GN \cdot MT = 0$
 $\leftrightarrow N^T G^T \cdot M T = 0 \quad \dots$ so if $G^T \cdot M = \text{Id}$, it works!
- We choose: $G = (M^{-1})^T$ (for orthogonal matrices, $G=M$)